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COVER PICTURE

The new Canadian Pacific liner "Empress of Britain", in which the G.E.C. has been responsible for the lighting of all the public rooms, involving the use of over two miles of Osram cold cathode tubing.

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Main hall of Errochty power station, North of Scotland Hydro-electric Board, where three 25 MW vertical-shaft alternators are installed

Automatically Controlled Hydro-electric Plant at Errochty Power Station

By J. W. LAING, M.I.E.E., and E. C. DAVIES, A.M.I.E.E.
Witton Engineering Works.

INTRODUCTION.

THE final stage in the construction of the Tummel-Garry Scheme of the North of Scotland Hydro-electric Board is approaching completion as the three 25 MW generating sets are brought into commission at Errochty, the third and largest power station in this scheme. As is well known, the scheme forms a major part of the overall plan for the harnessing of the waters of the River Tummel and its tributaries in the County of Perth. While the natural beauty of the area has been carefully preserved, the picturesque and historic valley of the Tummel has become a centre for the supply of electric power to a vast area extending from the industrial central belt of Scotland to the coastal regions of the North and East. It is estimated that the annual output from the six hydro-electric stations in the Tummel watershed will be approximately 600 million units, of which total the Errochty, Clunie and Pitlochry stations will together contribute about one half.

The original development of the hydro-electric resources of this area was put in hand in 1927, and resulted in the construction of the Rannoch and Tummel Bridge power stations. The former, which has an installed capacity of 48 MW, is supplied from Loch Erricht. A dam was built across the lower end of the loch and the waters of Loch Garry and Loch an-t-Seilich were diverted into this main reservoir. The water from the station is discharged into Loch Rannoch, which feeds the Dunalastair reservoir, formed by damming the River Tummel.

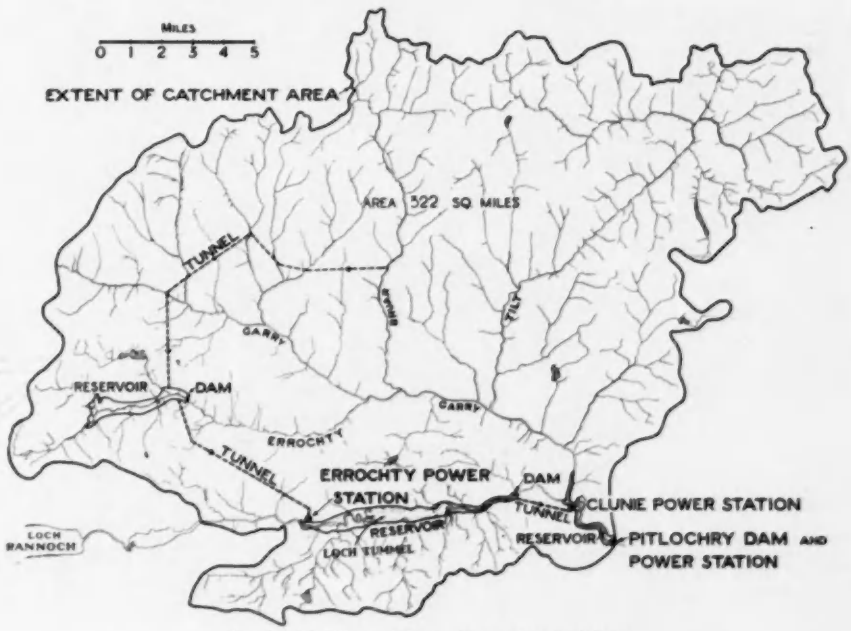

Fig. 1.—The Tummel-Garry Hydro-electric Scheme.

Fig. 2.—The diamond-headed buttress dam in course of erection in Glen Errochty.

From this reservoir is taken the supply to the 34 MW Tummel Bridge station.

THE TUMMEL-GARRY SCHEME.

Work on the Tummel-Garry Scheme (fig. 1) began in 1946. For this project, the total catchment area, most of which lies in the Grampians, is 322 square miles, the average annual rainfall ranging from 35 in. in the valleys to 120 in. at the higher altitudes.

The first stage in the programme involved the construction of the Clunie and Pitlochry generating stations, which are built downstream from the Rannoch and Tummel Bridge stations and re-use the water from them. About two miles downstream from the natural outlet of Loch Tummel, the Clunie dam was erected across the River Tummel, thereby raising the level of the loch by 17 ft. and more than doubling its length. From this dam, water is led through over two miles of tunnel and pipeline to the 61·2 MW Clunie station, which stands near the point where the Garry flows into the Tummel, on the shore of Loch Faskally. This loch is, in fact, a reservoir formed by the Pitlochry dam, two miles further downstream near the town of Pitlochry. The associated generating station, which is integral with the dam, has an installed capacity of 15 MW and is controlled from the Clunie station. Both stations in this section of the scheme were in full operation by the end of 1950.

ERROCHTY POWER STATION.

The Errochty project, with which this article is primarily concerned, was put in hand in 1948. The catchment area for this section is 86 square miles, in which the average annual rainfall is 55 in. The main reservoir, in Glen Errochty, is the highest of the three in this scheme, having a top water level 1,080 ft. above sea level. The diamond-headed buttress dam (fig. 2), which has been built to form this new loch, is 127 ft. high and 1,310 ft. long. Water is collected from the River Errochty and also, through a tunnel 12 miles long, from the upper reaches of the Rivers Bruar and Garry and some of their tributaries. From the reservoir, a tunnel 15 ft. in diameter leads the water six miles to the Errochty generating station, which stands on

Fig. 3.—The Errochty power station.

the shore of Loch Tummel near its western end. After passing through the station, the water is delivered to this loch and, therefore, joins the flow from the Tummel Bridge station to increase the annual output of Clunie.

The power station building (fig. 3), which is faced with stone excavated during the driving of the main tunnel, is constructed on three main levels, as shown in fig. 4, with a concrete raft at the back of the station on which the transformers are mounted. The main hall (see frontispiece) measures 122 ft. long by 50 ft. wide. The station houses three 25 MW generators and is designed for completely automatic operation by remote control from the Group Control Centre near Loch Faskally, alternative facilities being provided for local automatic control from the station itself and for local manual control. The alternators, a number of auxiliary driving motors, and the majority of the electrical control gear were supplied and installed by the G.E.C. It is estimated that the annual output of the station will be 103 million units.

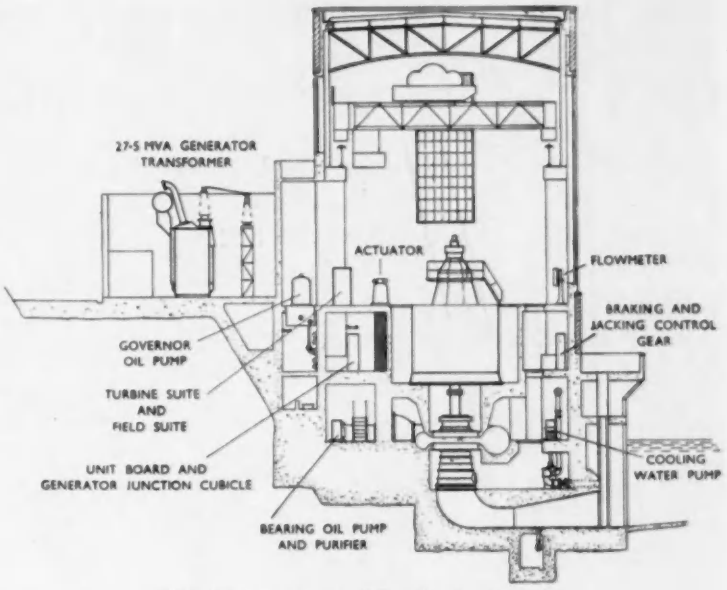

Fig. 4.—Cross-section of Errochty power station.

GENERATING PLANT.

As stated above, the total capacity of the station is 75 MW, the generating plant comprising three identical vertical shaft alternators, each coupled direct to a 35,000 h.p. Francis turbine of Boving manufacture. The alternators are designed to generate at 11 kV, 0·9 p.f., when driven at 428 r.p.m., the complete sets having a safe overspeed of 810 r.p.m.

ALTERNATOR STATOR.

The stator frame of each alternator (fig. 5) is octagonal in shape and is fabricated in two parts which are bolted and dowelled together. The segmental core stampings are secured by dovetailed keys bolted to longitudinal ribs inside the frame.

The winding is of the two-layer diamond type, the connections being brought out through an insulating plate near the bottom of the frame. Class B insulation, consisting of mica and micanite, is used throughout. The end windings are protected by aluminium guards which also assist in guiding the cooling air through the machine.

Fig. 5.—The stator of one of the three 25 MW hydro-alternators.

ALTERNATOR ROTOR.

The rotor shaft and coupling are forged in one piece, the rotor body, comprising two forged steel

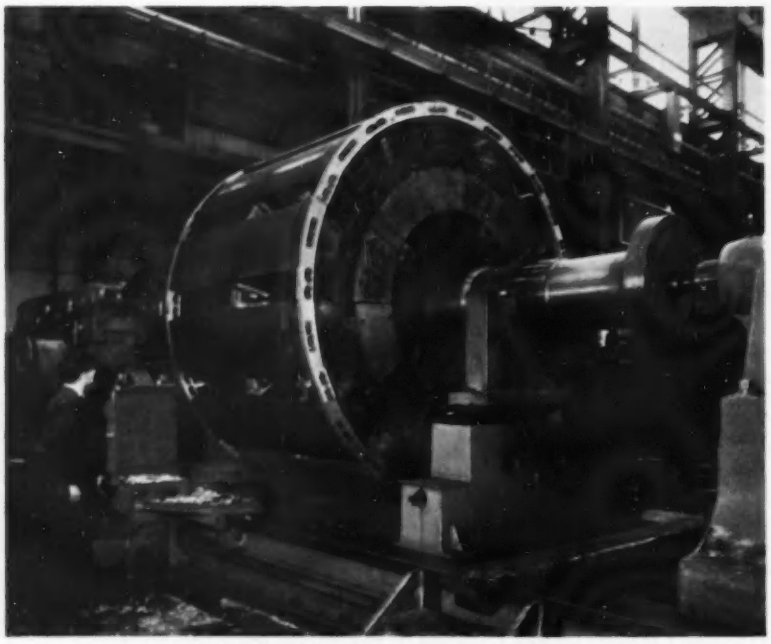

Fig. 6.—The rotor of one of the alternators undergoing final machining.

rims, being shrunk on and keyed to a cast-steel spider which, in turn, is keyed to the shaft (fig. 6). The laminated poles are dovetailed into the body and secured by taper keys. The rotor is supported in the top bracket by a combined thrust and guide bearing of the Michell type, designed to carry a total load of 180 tons. This bearing is insulated from the supporting structure in order to eliminate circulating currents in the rotor shaft. The bottom guide bearing is of the sleeve type and is of G.E.C. manufacture.

EXCITER ASSEMBLY.

The armatures of the main and pilot exciters are mounted on a forged steel shaft which also carries the cupro-nickel alternator sliprings and is solidly coupled to the alternator shaft. The main exciter is separately excited, while the compound-wound pilot exciter is provided with a shunt field regulator for initial adjustment of its field current. The permanent-magnet alternator for supplying power to the turbine governor motors is flexibly coupled to the exciter shaft, and above this are mounted the speed-limiting device and tachometer. Since the exciters are enclosed, bulkhead lighting fittings are provided to illuminate the interior of the housings so that the commutators, sliprings and brushgear can be observed through inspection windows. A raised platform surrounds the exciter assembly and access to the brushgear for maintenance purposes is provided by hinged doors in the housings.

VENTILATION.

The cooling system forms a closed air circuit, cool air being drawn from the alternator pit and forced through the machine by means of two centrifugal fans, one at each end of the rotor, and returned to the pit through the cooler. Cooling air for the exciters is diverted from the main supply, circulated through ducts, and returned to the pit through two sections of the cooler. Dampers are provided in the floor to divert warm air for station heating. Compensating air, to make up any

Fig. 7.—One of the 25 MW alternators on the test bed, showing arrangement of coolers.

losses in the system, is drawn through a filter in the wall of the alternator chamber.

The cooler, which is of the closed-air-circuit type manufactured by the Premier Cooler and Engineering Co. Ltd., comprises eight self-contained, water-cooled sections mounted around the shell of the machine as shown in fig. 7. The capacity of each section is such that any seven units are capable of dissipating the full load losses, so that any one section can be isolated for maintenance while the machine is at full speed. Six of the sections have a capacity of 7,260 cu. ft. of air per min., and the other two a capacity of 8,220 cu. ft. per min. The two larger units allow for the air flowing from the exciters through the ducting. Four of the remaining six sections are fitted with dampers to permit air to be by-passed into the station, while the other two are used purely for cooling the air from the alternator.

Cooling water, which is drawn from the tailrace by vertical spindle centrifugal pumps driven by 20 h.p., 415 volt, 1,450 r.p.m. squirrel cage motors (fig. 8), is fed to the coolers from a ring water main at the top of the alternator pit. With a water inlet temperature of 18 deg. C. and a rate of flow of 616 gallons per min., the complete cooler will reduce the temperature of 60,000 cu. ft. of air per min. from 50 deg. C. to 27 deg. C.

LUBRICATION.

The alternator guide and thrust bearings are arranged for flow lubrication, oil from a 650-gallon tank being circulated by means of a multi-stage pump having a capacity of 78 gallons per min. at 100 lb. per sq. in. pressure. This pump is driven by

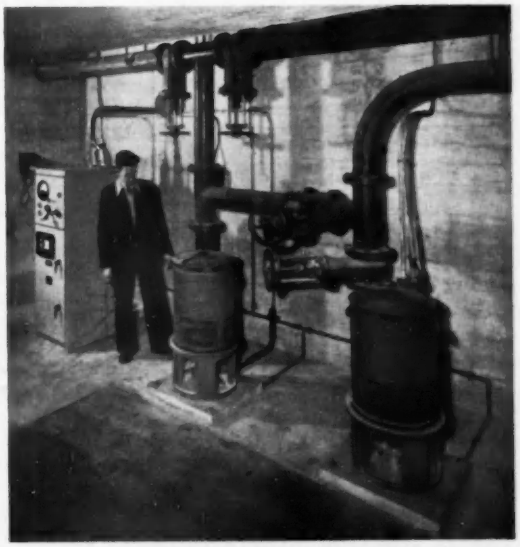

Fig. 8.—20 h.p. motor for cooling water pump of No. 2 set, and standby pump motor serving all three sets.

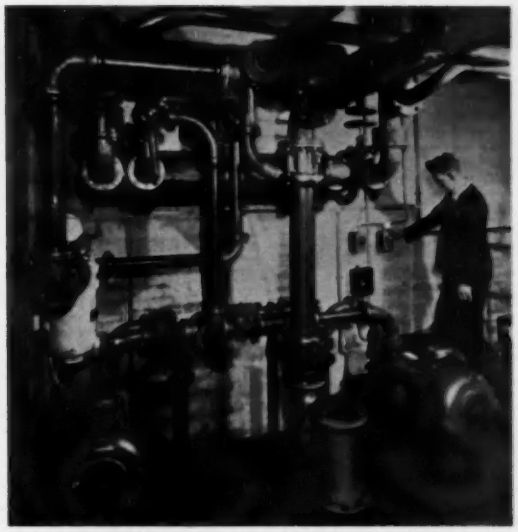

Fig. 9.—Main and standby bearing oil pumps with 11 h.p. motors for one of the sets.

an 11 h.p., 2,900 r.p.m. squirrel cage motor (fig. 9). A similar standby pump, the driving motor of which is fed from an independent source, is automatically brought into operation if the rate of oil flow falls below a preset value. The oil circuit includes an oil strainer and a cooler capable of dissipating 18,000 B.Th.U. per min., and designed to handle 165 gallons of oil per min. A standby cooler is provided to allow for servicing while the set is in operation.

BRAKING AND JACKING SYSTEM.

The braking and jacking equipment (fig. 10), which was supplied by Jones, Tate & Co. Ltd., of Bradford, comprises six rams, equally spaced and acting upon a brake ring secured to the base of the rotor, the top of each ram forming a flat plate which carries a heavy brake lining. Although the braking and jacking systems are interlinked, they are operated independently.

In the braking system, oil is forced from the reservoir by air fed into the top at a pressure of 100 lb. per sq. in. This air pressure is produced by means of a compressor (fig. 11), a single machine performing this duty for all three alternators. It is driven by a 2 h.p., 415 volt, 1,450 r.p.m. motor. An auxiliary pump is provided for use in the event of compressor failure. The oil passes from the reservoir, through a strainer and a non-return valve, to the circle of cylinders, forcing the rams upwards against the brake ring. This action provides a braking torque which is capable of arresting the motion of the rotor in about 2½ min. The scheme is such that the brakes cannot normally be applied until a rotor speed below normal is reached. Due to the low pressure of air

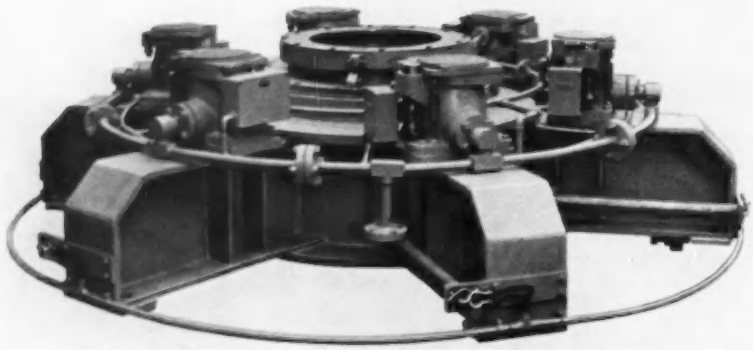

Fig. 10.—Rotor braking and jacking equipment, showing the six rams.

in the reservoir, the force applied to the rams by this system is insufficient to lift the rotor.

The jacking system is provided to lift the rotor, when at standstill, to allow an unrestricted flow of oil through the thrust bearing pads before starting after a prolonged shut-down; and also to allow maintenance to be carried out. Oil is fed direct to the cylinders at a pressure of 1,500 lb. per sq. in. by means of an Electraulic high-speed multi-ram pump (shown in fig. 12), which is driven by a 1 h.p., 415 V., 1,450 r.p.m. motor. Limit switches on the cylinders restrict the upward travel of the rams and ensure that each one rises to exactly the same height. If these switches should fail, mechanical stops come into operation, and, in this case, the excess oil supplied by the pump is returned to the supply tank through the high-pressure, solenoid-operated relief valve. Electrical interlocks ensure that the machine can be started only when all rams are in the fully-lowered position.

Hand-operated sustaining screws are provided to support the rotor in the elevated position, mechanical locking gear being incorporated to prevent these devices from being raised or lowered until the rams have been elevated. This arrangement also prevents the screws from being engaged when the rams are being used for braking, as it is necessary to lift the rotor slightly, even with fully-worn brake linings, before the locks are cleared.

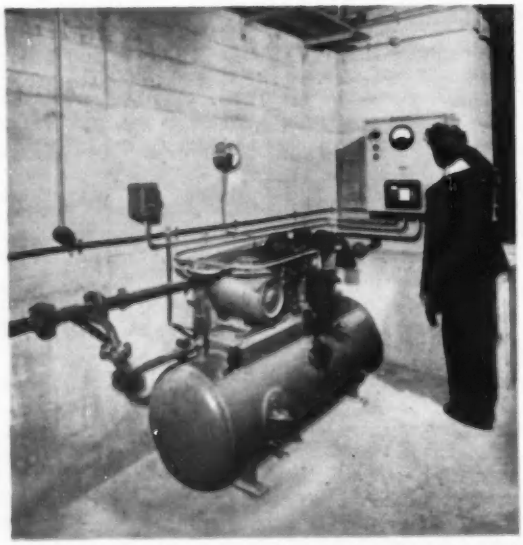

Fig. 11.—Brake air compressor for the three alternators, with 2 h.p. motor and starter.

Turbines.

The Francis turbines, which operate under a net head ranging from 492 to 575 ft., are, in the main, of standard design. The runner, in each case, is a one-piece stainless steel casting having a nominal diameter of 6 ft., and is solidly coupled to the turbine shaft, which is guided, at its lower end, by a white-metal-lined sleeve bearing lubricated with oil obtained from the alternator lubrication system. The water flow through the runner is controlled by the movement of the guide vanes, each vane being connected by a lever and link to the regulating ring. The movements of this ring are dictated by the main servomotor, which is controlled by the turbine governor, arrangements being made to lock the guide apparatus when the vanes are in the fully-closed position.

Each set is provided with a 5 ft. 6 in. dia. main inlet valve of the rotary type with a by-pass sluice valve. Both valves are hydraulically operated and are controlled by means of a solenoid-operated combined by-pass operating valve and main pilot valve. Limit switches, for interlocking and indicating purposes, are fitted at each end of the main valve travel. An automatic relief valve is also fitted. This valve opens immediately the main servomotor closes, and it then closes at a controlled rate such as to prevent an excessive rise of pressure in the pipeline.

Turbine Governing Gear.

The nerve-centre of the governing system for each turbine is an actuator (fig. 13) in the head of which is mounted a sensitive speed governor. The pendulum of this governor is driven by a small synchronous motor supplied from the permanent-magnet generator coupled to the alternator exciter shaft.

The movements of the governor pendulum with changes of turbine speed are transmitted, by means of a small auxiliary servomotor in the actuator base, to a regulating valve on the main turbine servomotor. This valve controls the admission of oil to the servomotor piston so as to produce the required movement of the guide vane regulating ring. Facilities for hand-control are also provided. A compensator is fitted to prevent over-travel of the guide vanes and consequent hunting.

Speeder gear, for controlling turbine speed when synchronising, is mounted on the actuator and may be adjusted by hand or by means of an electric motor. A load limiter, which varies the amount by which the main servomotor, and therefore the guide vanes, can open, is also provided on the actuator. This device may be set, either by hand or by a motor drive, for any guide vane opening from the closed to the full-load position.

Governor oil is supplied by means of an electrically driven pump delivering into a receiver, the upper part of which is filled with compressed air so that sufficient energy is available to ensure quick governor movement. The volume of air is automatically maintained constant by an oil-operated compressor. An auxiliary oil pump, driven by a Pelton wheel, is automatically started if the pressure should fail, and a standby compressor, also with Pelton wheel drive, is provided for charging the receivers when no external supply is available.

Drainage and Dewatering Equipment.

A horizontal centrifugal drainage pump is provided for removing any water which may collect in a sump at the lowest part of the station. This pump is driven by a 2 h.p. squirrel cage motor and is automatically controlled by the level of water in the sump. A similar pump, driven by a 5 h.p. motor, is used for dewatering the draft tubes so that access may be obtained to the underside of the turbine runners.

CONTROL SYSTEM.

As already stated, three alternative methods of control are available. The station may be operated automatically either by remote control from the Group Control Centre or by local control from the station itself, and facilities for local manual control are also provided. The method of operation to be employed is selected by means of "auto-manual" and "local auto-remote auto" control selector switches.

The main items of electrical control gear for each set are housed in a turbine suite and a field suite (fig. 13) which are mounted on the main floor adjacent to their associated generators; and in a unit board (fig. 14). The latter is installed, together with a generator junction cubicle in which the main control connections are marshalled, in the alternator chamber on the floor below.

The centre from which the station is operated under local automatic control is the turbine suite, the turbine panel of which incorporates all the controllers, meters, gauges and thermometers required for this purpose, together with a comprehensive system of indicators and alarms. In addition, the majority of the associated relays and contactors, including interposing relays through which remote control is effected, are mounted in the turbine contactor panel of this suite. The field suite houses the automatic voltage regulator with the whole of its associated equipment.

Starters for the motors driving the governor oil pump, bearing oil pump and cooling water pump are mounted in the unit board, the 415 volt, 3-phase supplies being drawn from the corresponding unit transformer. The standby bearing oil pump motor

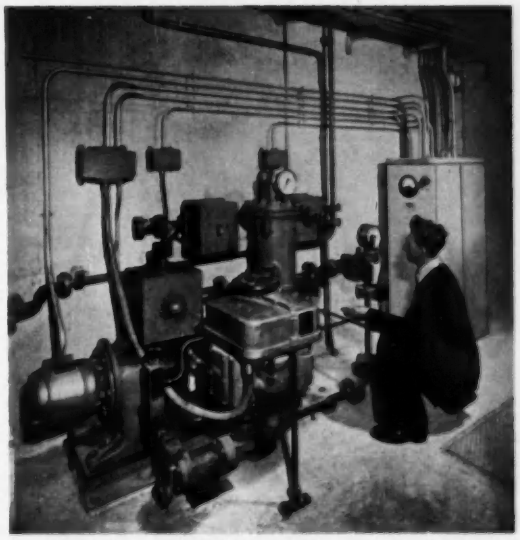

Fig. 12.—Jacking and braking control equipment for one set, showing 1 h.p. jacking oil pump motor and control cubicle.

is also controlled from the unit board, but its starter is connected to the common services board, the supply to which is independent of the unit transformer so that the pump can be operated even when none of the sets is running. In addition to the equipment already mentioned, separate starters are provided for the remaining auxiliary drive motors, and these also are supplied from the common services board.

Overall Controls.

Once the field switch has been closed, the operation of each set is under the overall control of a master control relay, housed in the turbine contactor panel. When the selector switches mounted on the turbine panel are set to the MANUAL position, this relay is energised, by means of a limit switch, as soon as the guide vanes are unlocked. Under local automatic control the relay is closed by the START push button, and under remote control by contacts on an interposing relay. Contacts on the master relay control the energisation of Start and Stop buswires and,

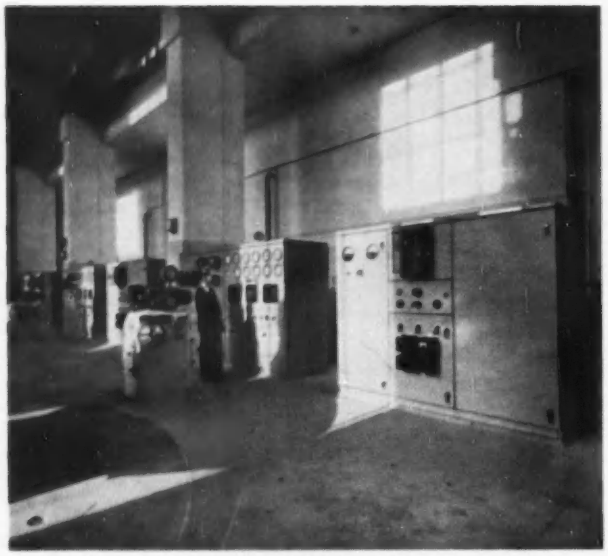

Fig. 13.—Field suites and turbine suites, with actuators for sets Nos. 2 and 3.

thereby, the automatic operation of the complete set.

Emergency shut-down is initiated either by means of the turbine shut-down relay, which operates under certain alarm conditions, or by a timer which is arranged to operate if the time taken to bring the generator up to speed with all auxiliaries operating correctly should exceed a predetermined value.

Main Inlet Valve Control.

Manual control of the main inlet valve is effected by means of a controller on the turbine panel, but under automatic control the valve is opened or closed by energisation of the Start or Stop buswire respectively. A limit switch on the guide vane lock is connected in series with the valve-opening solenoid and prevents the valve from being opened, under either manual or automatic control, unless the vanes are locked. A similar device ensures that the valve can only be closed automatically when the guide vanes are closed. Closure of the valve may also be prevented, either locally from the turbine panel or remotely from the Group Control Centre, by means of a lockout relay, but, in an emergency, operation of the turbine shut-down relay will close the valve under either control and independently of the lockout.

As soon as the inlet valve begins to open, a time delay relay is energised. If the valve functions normally, the relay is de-energised at the fully-open position, but if, for any reason, the time required for the complete travel is excessive, the relay operates to isolate the valve control solenoids from all automatic control, thus ensuring that any further movement of the valve is the result of a deliberate manual action.

Guide Vane Control.

The Start and Stop push buttons by which the guide vanes are operated under manual control are mounted on the actuator, automatic operation, once again, being dependent on the energisation of the Start and Stop buswires. A system of limit switch and relay interlocks ensures that, before the vanes can be opened, the rotor jacks must be fully lowered with the sustaining screws withdrawn, the load limiter must be in the fully closed position and the inlet oil to the main servomotor must be at its normal pressure. Under automatic control, it is further ensured that an adequate supply of lubricating oil is flowing to the bearings and the main inlet valve is open before the guide vanes are opened, and the actuator must be in the closed position before the vanes can be closed. The shut-down relay, however, as in the case of the main inlet valve, exercises overriding control.

A meter on the turbine panel is energised by means of an auxiliary relay to record the time that the guide vanes are open.

Jacking and Braking Control.

The control circuits for the jacking and braking equipment are so arranged that neither operation can be carried out unless either the guide vanes are locked or the main inlet valve is closed. In addition, an interlock on the master control relay ensures that the starting sequence has not been initiated.

The jacking operation can only be performed under manual control from the turbine panel. Move-

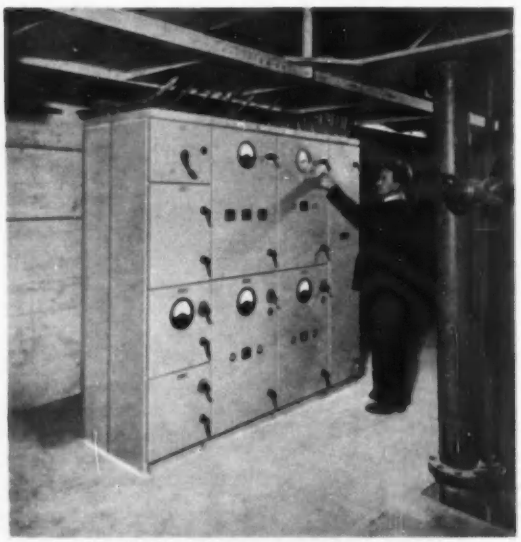

Fig. 14.—Unit board and generator junction cubicle for one of the sets.

ment of the controller to the RAISE position energises a motor which closes the regulating valve on the jacking gear. Immediately the valve is closed, the jacking pump starts automatically and raises the jacks until, at the fully raised position, a limit switch operates to cut out the jacking pump motor. Alternatively, the pump may be stopped in any jacking position by means of a switch on the turbine panel. When the controller is set to LOWER, the regulating valve is driven to a " restricted open " position so that the rotor is lowered gradually until it rests on the thrust bearing, thus relieving the fluid pressure in the system. As a result, contacts on the pressure gauge operate to energise the driving motor of the regulating valve, which moves the valve to the fully open position.

The brakes are applied by admitting compressed air, through the brake valve, to the oil reservoir, which is closed to atmosphere by means of another valve. Under manual control from the turbine panel, movement of the controller to the ON position closes the atmosphere valve, after which contacts on the tachometer operate as soon as the machine is running at less than 50 per cent. of the normal speed, closing the jacking regulating valve and opening the brake valve so that the brakes are applied. Under automatic control, the brake valve solenoid is energised by means of an auxiliary stopping relay. If the turbine shut-down relay operates, the brakes are applied as long as the set is running at less than 107·5 per cent. normal speed. The brakes may be released manually by means of the controller or automatically by a time delay relay which operates after the brakes have been applied for a preset time of up to 5 minutes. After emergency braking, the shut-down relay must be reset. Either of these operations closes the brake valve and opens the jacking regulating valve, first to the restricted and then to the fully open position.

EXCITATION.

The field-switch is hand-operated but is automatically tripped by the operation of the turbine shut-down relay.

VOLTAGE REGULATION.

The automatic voltage regulator is of the vibrating contact type, the moving contacts being vibrated, by means of a motor-driven cam, between two adjustable fixed contacts. The position of these fixed contacts relative to the vibrating contacts is controlled by the operating coil. Control of the generated voltage is sectionalised to give coarse and fine regulation, and also to provide more precise excitation control when the voltage deviation is in the region of 5 per cent. Facilities are also provided enabling the alternator to be run up or shut down automatically under the control of the regulator. Compensation for parallel running is provided by the quadrature droop method, the necessary current compounding being injected into a separate winding on the operating coil. The equipment is also compensated for line drop, the components of the compensating current proportional to the wattful and wattless components of the line drop being summated by means of a mixing transformer which is common to all three sets.

The operating coil is provided with a frequency-compensated, positive phase sequence supply. No-volt protection is afforded by a specially designed relay which discriminates between low voltage due to a system fault and low voltage due to a failure of the control circuit, thus preventing forcing of the excitation if the reference voltage should fail. Power for the regulator operating equipment is normally supplied from the station battery, but the connections are automatically transferred to the pilot exciter when the set is running and synchronised.

Fig. 15.—Starters for dewatering and drainage pump motors, with cooling water pump motor for one set in foreground.

The generated voltage may be adjusted independently of the automatic regulator by direct control of the driving motor of the exciter field rheostat, either locally by means of a controller on the field suite, or remotely from the Group Control Centre by an interposing relay.

SPEEDER MOTOR CONTROL.

The driving motor for the speeder gear on the actuator can be operated under either manual or local automatic control from the turbine panel, or, alternatively, under remote automatic control by means of an interposing relay. Under all control conditions, the motor circuit is broken by limit switches at the maximum and minimum settings.

Load Limiter Motor Control.

The motor drive for the load limiter is used only under automatic control. When it is desired to raise the load, the motor is energised from the Start buswire, provided that the guide vanes are unlocked. The limiter is then driven until it reaches the fully open position, at which point the circuit is broken by a limit switch. The supply for lowering the load is taken from the Stop buswire, and a further limit switch operates when the load limiter is fully closed.

Motor-driven Pump Controls.

As already stated, the main pumps for governor oil, bearing oil and cooling water are controlled by starters housed in the unit board. The contactors are of the latching type and are operated by means of Start and Stop push buttons, an Emergency Stop button being mounted near each motor. Since these machines are supplied from the unit transformer, they cannot be used during the starting-up period and, therefore, the standby pumps are used at this time.

The standby bearing oil pump motor, as explained earlier, while supplied from the common services board, is controlled from the unit board, normally through the medium of the Start buswire. Manual control from the turbine panel is possible for testing purposes. Under all conditions, the contactor operates under the control of a flow relay which starts the pump as soon as the flow falls below 4,500 gallons per hour. The standby cooling water pump can be operated under the control of push buttons on a separate starter cubicle or automatically by means of the unit no-voltage relay on any one of the three sets as determined by a selector switch on the starter.

The driving motors for the dewatering and drainage pumps are both provided with wall-mounted starters (fig. 15) and emergency trip buttons, but the drainage pump can also be operated automatically by means of a relay controlled from two electrodes, one mounted at high-water and the other at low-water level in the drainage sump. The brake air compressor motor can also be operated from its own starter or automatically under the control of a pressure switch. The control of the jacking oil pump has already been described.

Indicators and Alarms.

A comprehensive system of indicators, comprising lamps and flag relays, is provided both at the station and at the Group Control Centre. Local indication is given only under manual or local automatic control, the supply to the indicating buswire being disconnected when remote control is being employed.

Each set is provided with two ten-way alarm units fed through alarm type fuses, an indicating lamp being automatically illuminated if one of these fuses should blow. Each unit consists of telephone and flag type indication relays which are energised by various alarm contacts. Under manual and local automatic control, these units actuate an alarm bell and also a beacon lamp on the turbine panel of the set concerned, and for certain selected conditions, repeat alarms are initiated at the Group Control Centre.

TRANSMISSION.

The output of the station is stepped up to 132 kV and is transmitted by oil-filled underground cable to the Errochty switching station to which all the other stations in the Tummel Valley are connected. One 132 kV transmission line connects this station with hydro-electric schemes in Inverness-shire and Ross-shire and with distribution centres in the north-east, while a second forms a link with Loch Sloy and other schemes in that area.

Two further 132 kV lines carry power to the C.E.A. grid at Abernethy in Perthshire and at Bonnybridge in Stirlingshire.

ACKNOWLEDGMENTS.

Acknowledgments for assistance in compiling the above information and for permission to publish this article, are due to the North of Scotland Hydro-electric Board and to Merz & McLellan, who were the consulting engineers for the electrical equipment. The co-operation of Sir Alexander Gibb and Partners, the consulting engineers concerned with the civil engineering aspects of the scheme, is also gratefully acknowledged.

Lift Machinery Modernisation at Thames Tunnels

By W. T. WAGON
The Express Lift Co. Ltd.

Fig. 1.—South entrance to the Woolwich pedestrian tunnel and approach to the Woolwich Free Ferry.

EXCEPT among those who live in the neighbourhood, the pedestrian tunnels under the Thames at Greenwich and Woolwich are little known compared with the tunnels for vehicular traffic at Rotherhithe and Blackwall. The Greenwich tunnel, however, was opened in 1903, and that at Woolwich in 1912. Both works were undertaken by the London County Council to improve communications between the north and south banks of the river at a period of rapid increase in industry and population. From the time of their opening the two tunnels were provided with passenger lifts at both ends. All the equipment then installed, except the controllers, remained in service until late 1955. The work of replacement with modern variable-voltage drives was completed in ten weeks and the lifts, still retaining their original cars, resumed service last December. Fig. 1 shows the lift building at the south end of Woolwich tunnel, adjacent to the approach to the Woolwich Free Ferry.

The new machinery was supplied and installed for the L.C.C. by The Express Lift Co. Ltd., whose predecessor, The Easton Lift Co. Ltd., was responsible for the original installations at both tunnels. At the time of the opening, the Greenwich tunnel was the only crossing of the Thames between Tower Bridge and Blackwall, and its lifts were remarkable in their day for their size. A catalogue of the Easton Lift Co. Ltd. for 1905 lists them together with those supplied for the City & South London Railway which, with similar capacity, are described as "The largest Electric Passenger Lifts in the World". The number of passengers using the tunnels is indicated by the figures for 1954, when the number of journeys made by each of the lifts was approximately 227,000 at Woolwich, and 255,000 at Greenwich.

EARLIER CONTROL SYSTEMS.

As first installed the Greenwich and Woolwich lifts had the faceplate controllers usual for that period, but in the early 1930's the control apparatus at both

Fig. 2.—Original drive at south end of Woolwich tunnel.

tunnels was replaced by a camshaft system. This was the only change made in the installations until the replacement now completed.

The main controllers installed in 1932 included solenoid-operated switches for completing the main circuit, reversing the armature current and making the speed changes, but the insertion and cutting out of the resistances to obtain speed control was by means of a camshaft driven by a chain from the main driving gear through a friction clutch which enabled the shaft to be locked while the motor was operated at each speed.

On the Woolwich tunnel lifts operation of the car controller to " slow " set the main controller contacts in the appropriate direction, and when the motor started, the chain-driven camshaft closed a series of contacts which shorted out the armature resistance. The camshaft was then held stationary by a pawl until the car controller was moved to " fast ", which permitted a second series of cams to open contacts and insert resistances in the shunt field circuit which weakened the field and gave the final top speed. This operation was reversed on approaching the floor. Final stopping was achieved by switching off the motor, switching in a dynamic braking resistance across the armature, and applying the mechanical brake. Fig. 2 shows the original drive at the south end of Woolwich tunnel, and

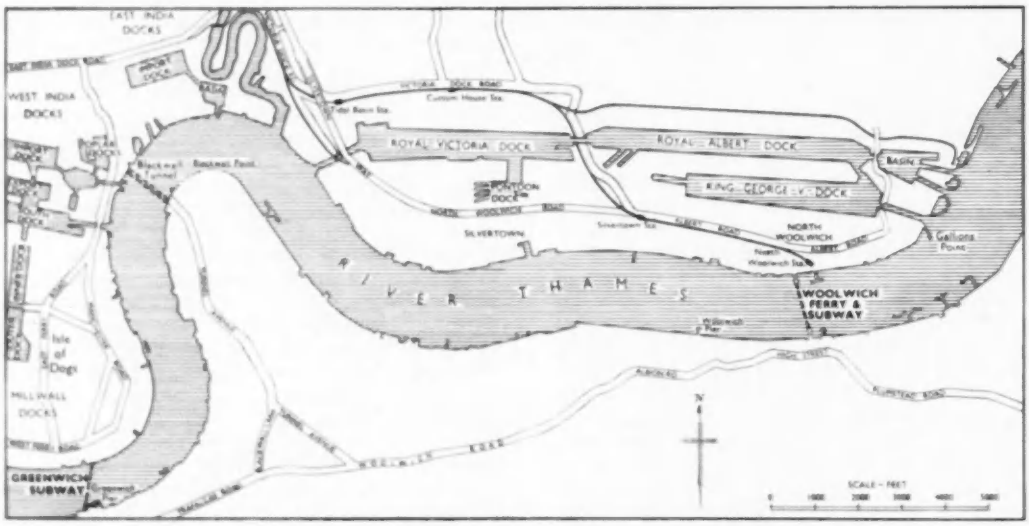

Fig. 3. —Location of the two tunnels in relation to the London docks.

the location of both tunnels is shown in fig 3. One of the 1932 controllers at Greenwich is seen in fig. 4.

On account of differences in motor characteristics, operation of the Greenwich tunnel lifts was slightly different from the Woolwich scheme. In general the control was similar to that above described, except that top speed was arranged without weakening the shunt field, and slow speed was obtained by reinserting the main starting resistance and then switching in a diverter resistance across the armature, these resistances being varied by the cam-operated contacts driven from the gear.

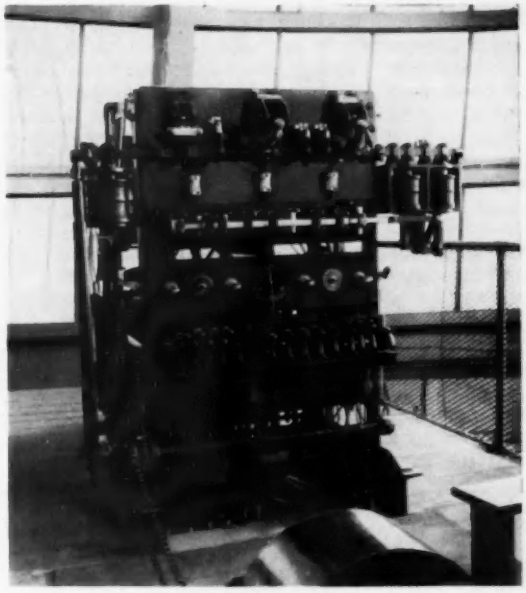

Fig. 4.—Camshaft controller installed at Greenwich in 1932.

THE "S" DRIVE AT GREENWICH.

Each lift was driven by a compound wound motor, of 64 b.h.p. for Greenwich and 50 b.h.p. for Woolwich. The traction drive to each car in the Greenwich tunnel was interesting in its period, in that it took the form of an "S" drive. It consisted of a driving sheave on which the eight 3 in. circumference ropes had a 270° wrap obtained by the positioning of an intermediate rope pulley, as shown in fig. 5. The driving sheave and the rope pulleys were all 5 ft. in diameter, and had "U" grooves for the ropes.

The drive for the Woolwich tunnel lifts was by means of a winding drum 5 ft. 6 in. in diameter. In order to obtain a satisfactory factor of safety, both the car and the counterweight were suspended on six $2\frac{3}{4}$ in. circumference ropes, but so as to keep the length of the drum to a minimum, only four ropes from the car and counterweight were wound on and off the spiral grooves provided, the remaining two ropes being connected between the car and a separate section of the counterweight, running above the main weight, and were carried over the drum by two circumferential grooves.

NEW VARIABLE VOLTAGE INSTALLATIONS.

Throughout its life the equipment at both tunnels had been supplied with D.C. by the Power Undertaking, although normal supplies in the area latterly were A.C. When it was decided that these installations had reached the end of their useful life, it was therefore convenient to arrange to use the available 3-phase supply. The new equipment is of the Ward-Leonard variable voltage type with generators driven by squirrel cage motors. Each motor-generator set at Greenwich consists of an 80 h.p. motor driving a 52 kW, 1,460 r.p.m., compound generator. The Woolwich sets are similar except that the ratings are 55 h.p. for the motors and 35 kW for the generators. The motor-generator sets are mounted on concrete blocks with cork pad sound insulation. Fig. 6 and 7 show the layout of the machinery at Greenwich and Woolwich respectively.

All lifts are now driven by 790/850 r.p.m., 230 volt, enclosed ventilated motors with ratings of 60 h.p. at Woolwich (fig. 8) and 40 h.p. at Greenwich (fig. 12). Each motor drives through a 57:1 worm reduction gear and a 39 in. diameter driving sheave connected to the worm wheel by a fabricated spider. A substantial brake with self-aligning shoes operates on a brake drum formed by the coupling between the motor and the worm shaft. The brake is applied mechanically by means of powerful springs, and is electro-magnetically released. In order to keep the effect of worm to worm-wheel wear to a minimum, the diameters of the sheave and the worm wheel have been kept as nearly as possible equal. The driving sheaves have preformed rope grooves to accommodate $\frac{7}{8}$ in. diameter ropes, of

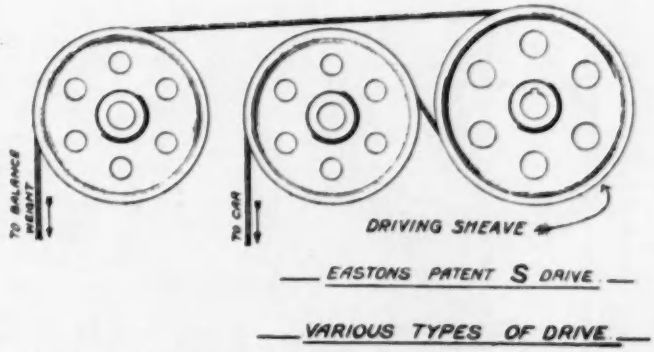

Fig. 5.—A diagram of the "S" drive, reproduced from a 1905 catalogue of The Easton Lift Co. Ltd.

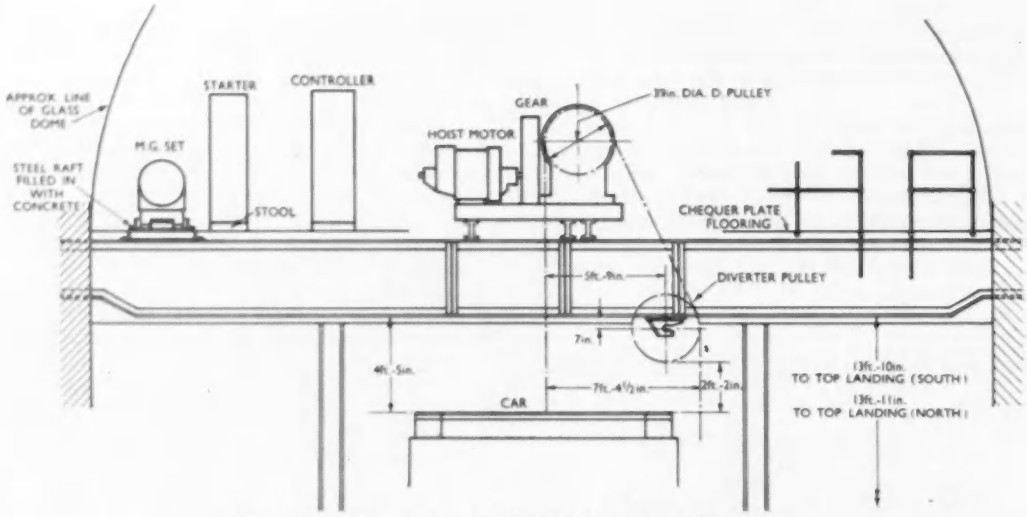

Fig. 6.—Layout of modernised installation for the Greenwich tunnel lifts.

which there are eight for each lift at Greenwich and six for each lift at Woolwich. Diverter pulleys, 39 in. dia., are provided to obtain the required suspension centres between the car and the counterweight. Epicyclic hand winding gear has been supplied for emergency use. When this is fitted to the worm shaft, all electrical circuits are broken automatically, and cannot be restored until the winding gear has been removed.

RENEWALS OF EQUIPMENT.

All the lift guides have been renewed. At Greenwich the car guides were machined from 85 lb. rail section and the counterweight guides from 50 lb. section. For the smaller lifts at Woolwich conventional machined T-section guides were employed. The original cars are still in use, and the existing governor-controlled safety gears have been overhauled and retained. The car and landing collapsible gates have been renewed, and new locks have been provided for the landing gates, fitted with pre-locking contacts which ensure that the gates must be not only shut but actually locked before the lift can be started.

A steel cubicle houses the starter for the motor-generator set, comprising a line contactor and an accelerating contactor, the latter closing to short circuit a stator resistance when the set approaches full speed. A separate cubicle accommodates the main controller with its contactors, relays, protective devices, resistors and 3-phase control rectifier and transformer, the latter providing 100 volts D.C. for all control equipment, brakes, and the fields of the generators and lift motors. The two cubicles at Greenwich are shown in fig. 9.

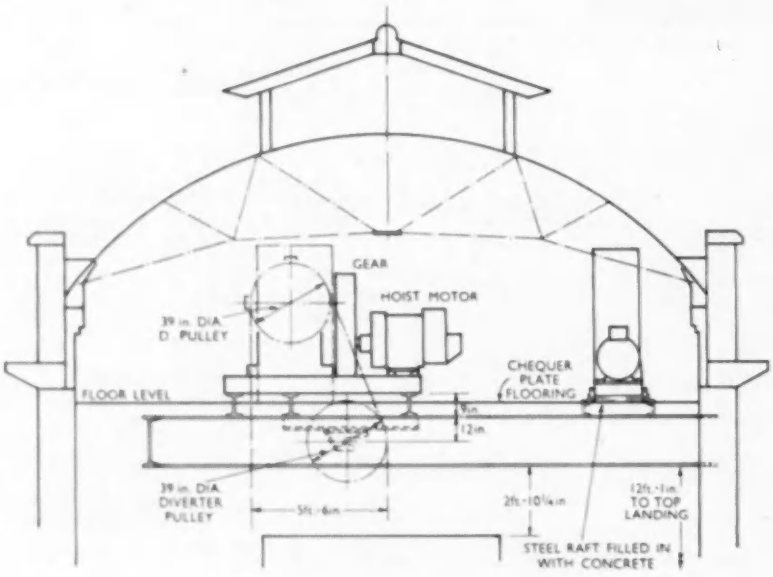

Fig. 7.—Layout of modernised equipment at Woolwich.

SEQUENCE OF OPERATIONS.

The control of the lifts, while still carried out by lift

attendants, has been changed from a car controller to push buttons. The control station in the car (fig. 10) includes " up " and " down " push buttons, key-operated switches for car light and remote control of the motor generator, and an isolation switch for disconnecting the car control. Below the car station is situated a distribution box with hinged cover in which the car wiring and the trailing flexibles are terminated. This arrangement makes them easily accessible from inside the car for maintenance and testing purposes.

On pressing the " up " or " down " push button in the car the hoisting motor field is restored to full strength, the generator armature circuit is closed and the generator shunt field is excited with the polarity necessary for the generator output to drive the hoist motor in the required direction. At the same time the brake lifts and the hoist motor starts.

Acceleration is effected by increasing the current in the generator shunt field by the closing of three time-controlled contactors, which short-circuit resistances in the 100-volt field excitation system; and finally top speed is attained by weakening the hoisting motor shunt field. As the car approaches the landings, limit switches in the shaft initiate deceleration by strengthening the hoist motor shunt field and reducing the generator shunt field excitation in two steps. The final levelling speed and the stopping of the car are then effected by magnetic inductor switches mounted on the car and operated by steel plates situated in the shaft. On passing the first inductor plates the shunt field excitation is reduced to its minimum value, and

Fig. 9.—Control and starter cubicles at north end of Greenwich tunnel.

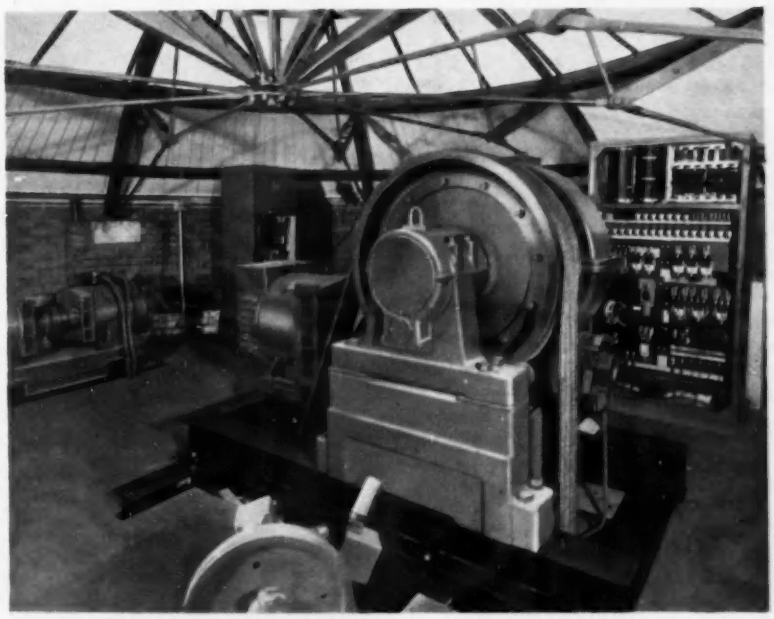

Fig. 8.—Machine room at Woolwich, north end, showing hoist motor, Ward-Leonard set and switchgear cubicles.

Fig. 10.—Control station in lift car at Greenwich tunnel, north end.

the car proceeds at levelling speed. On passing the second plate, the generator shunt field is disconnected, the differential field is brought into circuit, and the brake is applied to stop the car at floor level. Simultaneously the hoisting motor shunt field is reduced to standing value, and after about one second the generator-hoist motor circuit is opened by a single-pole contactor, thus ensuring complete isolation of the hoisting motor armature from the generator.

In addition to the switching described, control is exercised by the series winding of the generator. By switching diverter resistances across the series field, varied compounding characteristics are provided for top speed and levelling. This allows the deceleration brought about by regeneration in the loop circuit to be carried out in the minimum time, and ensures virtually constant final speed, enabling the car to stop accurately at floor level irrespective of the passenger load. This stopping is automatic and does not depend upon the skill of the lift attendant as previously.

The inherent characteristics of the new equipment, with its Ward-Leonard control, provide higher rates of acceleration and deceleration, and the higher average speed has increased the passenger handling capacity of the lifts.

Moreover, as it is now possible to control the rise and fall and the direction of very heavy currents in the lift motor by varying the relatively small currents in the generator shunt field, the new switchgear is of a much lighter type. This means that the wearing parts are smaller, and replacements will be at a slower rate than was necessary with the heavier type of switchgear.

MAINTENANCE FACILITIES.

On the original equipment no facilities were provided for the maintenance mechanic, apart from test pushes on the control panels, but with the new control a " mechanic control station " is located on top of the car, and contains " up " and " down " pushes, switches for test and roof light, and an emergency switch. A socket outlet is also provided. The test switch brings the " up " and " down " pushes into circuit, and at the same time disconnects the normal car control. The car will travel at slow speed in the appropriate direction only while pressure is maintained on the push, always provided that all safety circuits through locks and other devices are completed. An additional limit switch is provided to stop the car approximately 3 ft. below the top level so as to prevent personnel travelling on top of the car from coming into contact with the overhead steelwork. The emergency switch provides overall protection and the socket outlet allows for an inspection lamp or portable tool to be used on top of the car.

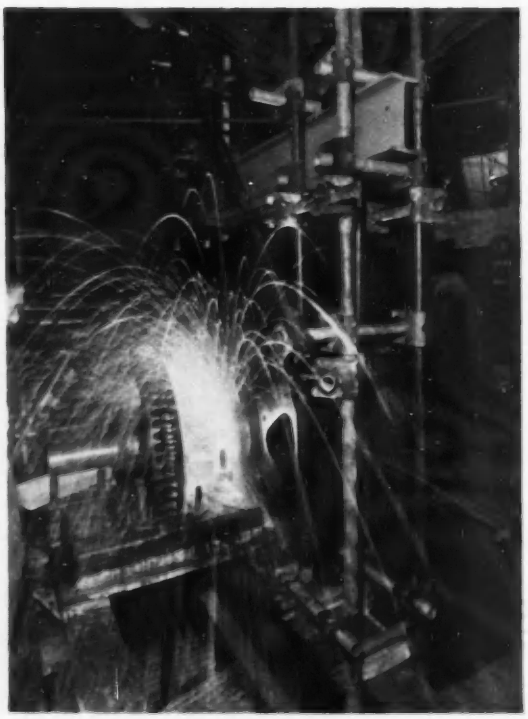

Fig. 11.—Flame-cutting a drum shaft in order to remove old drum at Woolwich tunnel.

INSTALLATION WORK.

Removal of the existing heavy equipment, and installation of the new machinery, introduced some difficult handling problems, and tubular scaffolding with steel joists for the lifting tackle was provided at all shafts by the L.C.C., and was designed for a weight of approximately 20 tons. At Greenwich the lift equipment, as it was dismantled, had to be traversed sideways before being lowered, as it was only near the doors into the lift buildings that the floors would bear the weights involved. By flame-cutting the bedplate of the gear into two pieces while still in position, the heaviest lift was restricted to approximately 5 tons. Other parts were also cut while in position. The work of dismantling and erection was completed and the lifts were ready for passenger service in ten weeks.

At the Woolwich shafts the space was more restricted, and after the scaffolding had been erected in suitable positions for removing the old equipment, it had to be moved for the installation of the new. It then had to be dismantled before the control cubicles could be placed in position. In order to remove the drum, the drum shaft had to be flame-cut on both sides (fig. 11). The drum was then lowered and handled out through the top landing gates before being set down on the floor. As at Greenwich, the gear bedplates were flamecut in position into two parts in order to limit the maximum weights handled.

Fig. 12.—Variable-voltage equipment for lift at north end of Greenwich tunnel.

ACKNOWLEDGMENTS.

All the work was carried out under the direction of Mr. J. Rawlinson, C.B.E., M.Eng., Chief Engineer of the L.C.C., assisted by Mr. C. Belcher, M.I.E.E., and Mr. C. G. L. Morley, A.M.I.E.E., A.M.I.Mech.E. We are indebted to Mr. Rawlinson for permission to publish this description and for supplying the illustration (fig. 4) of the earlier camshaft controller at Greenwich.

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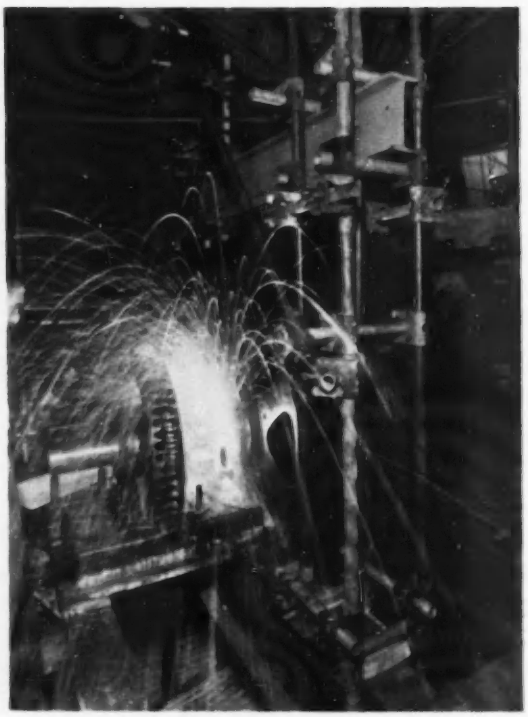

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The Critical Speeds of Rigidly-coupled Rotors

By J. M. MITCHELL, B.Sc.(Eng.), D.I.C., A.M.I.Mech.E.,
Chief Engineer, Turbine Department, Fraser & Chalmers Engineering Works.

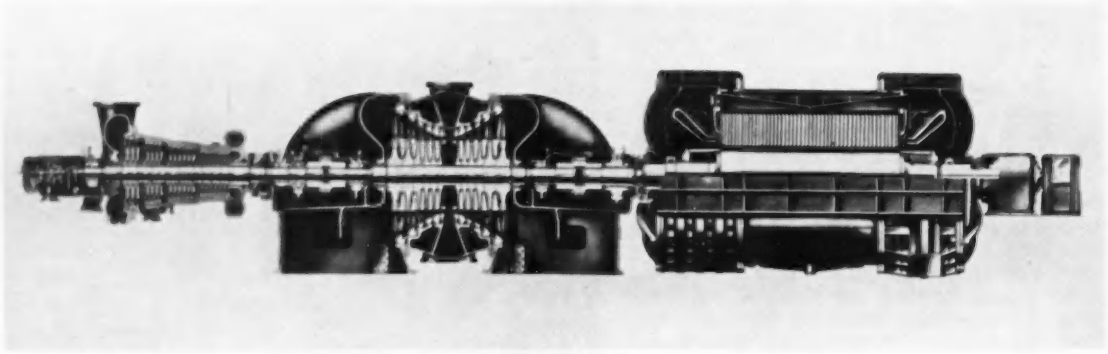

Fig. 1.—Longitudinal section of 60 MW turbo-generator.

INTRODUCTION.

UNTIL recent years the rotors of steam turbo-generators were usually connected by means of flexible couplings, the purpose of which was to permit slight misalignments and absorb differential expansions, each rotor having normally its own thrust bearing. With increasing unit ratings, the torques transmitted became very large and the ability of the flexible couplings to absorb misalignment became a matter of some doubt since the rotors were virtually locked together under load. With careful alignment this difficulty was readily overcome, but the large axial forces necessary to produce relative sliding of the coupling elements during accommodation of differential expansions while under load often caused failure of thrust bearings.

A very satisfactory solution is to couple the rotors rigidly together, using one thrust bearing only, and to design the machines so that the effect of cumulative differential expansions is minimised. Fig. 1 shows a G.E.C. 60 MW. turbo-generator of current design in which this principle is utilised, the thrust bearing being between the H.P. and L.P. turbine rotors. This casing arrangement results in the rotor-casing differential expansion of H.P. and L.P. rotors being virtually the same as if each had its own thrust bearing while the generator is readily able to accommodate the axial shift of its rotor produced by the small expansion of the L.P. turbine rotor.

It is interesting to note that the G.E.C. was the first British maker to institute the use of rigid couplings on a large scale. The principle is now well established and it can fairly be claimed that the use of rigidly-coupled rotors makes a most important contribution to the satisfactory operation of steam turbo-generators having the very large ratings now required.

The problem of ensuring that the critical speeds of the rotors are sufficiently far removed from normal running speed becomes more difficult in the case of rigidly-coupled rotors than when flexible couplings are employed, since the latter possess a degree of detuning and damping, even while transmitting heavy torque, sufficient for the combined critical speeds of the various rotors to be of no importance. For design purposes a knowledge of the critical speeds of each rotor independently was therefore sufficient. With rigid coupling the combined critical speeds must be considered very carefully and the purpose of the following brief account is to outline the methods used for their calculation in the Turbine Design Department at Erith, some of which were developed by the staff of that department and are believed to be original.

For a simple rotor on two bearings, many well-

known methods exist for the calculation of critical speeds, especially for the first critical speed, which is generally of greatest importance. For rotors of normal proportions most of these methods give closely similar results and choice of method may be a matter of individual preference. In the case of continuous rotor systems carried on three or more bearings, and even for the higher critical speeds of two-bearing systems, many of these methods fail, especially those which depend on the calculation of the static deflection due to the weight of the rotor.

THEORY OF THE STODOLA METHOD.

The author's Company has always used the method of Stodola[1] for two-bearing rotors and since this is the basis of its methods for more complex systems an outline of the theory is given.

The following notation is used, other symbols being defined in the text :

- a Constant.
- A Area under bending moment diagrams.
- E Young's modulus for shaft material.
- f Functional symbol.
- g Gravitational acceleration.
- I Moment of inertia of shaft cross-section.
- l Length of shaft.
- M Bending moment on shaft.
- t Time.
- w Loading on shaft per unit length (rate of loading).
- x Distance of point on shaft from suitable origin.
- y Displacement of point on shaft centre-line from static position.
- ω Angular velocity (radians per second).

Neglecting gyroscopic effects due to radial dimensions of the wheel discs, etc., the differential equation for the radial displacement due to whirling at a critical speed is obtained from the fact that the rate of loading at any point is equal to the centrifugal force, that is

$$\frac{d^2M}{dx^2} = \frac{w}{g}\omega^2 y$$

and since $M = EI\frac{d^2y}{dx^2}$ this becomes :

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2y}{dx^2}\right) = \frac{w}{g}\omega^2 y \qquad (1)$$

where in general I and w vary with x.

The solutions are

$$y=f_1(x), \qquad \omega=\omega_1$$
$$y=f_2(x), \qquad \omega=\omega_2$$
$$y=f_n(x), \qquad \omega=\omega_n$$

where the various values of ω are the Eigen values corresponding to each solution and $f_1(x)$, $f_2(x)$...etc., are the equations of the deflected forms assumed by the shaft centre-line under the centrifugal loading.

If $\omega_1, \omega_2, \ldots\omega_n, \ldots$ are in increasing order of magnitude they correspond to the angular velocities at the 1st, 2nd,...nth...critical speeds.

The functions $f_n(x)$ have the property that if $y=f(x)$ is a solution, then $y=af(x)$ is also a solution where a is any constant. This follows from the linearity of equation (1), centrifugal force and elastic restoring force both varying linearly with y.

On account of the variation with x of I and w, direct analytical solutions of equation (1) are possible in only a limited number of cases.

In the Stodola method, we first assume a deflected form which is consistent with the end conditions and an angular velocity ω_o. From the assumed deflected form and ω_o we can readily calculate the centrifugal loading and hence in turn the bending moments and the corresponding deflected forms. The well-known graphical method using the funicular polygon is most convenient for these calculations in view of the variation of I and w.

In general, the deflected form y_1, corresponding to the centrifugal forces of the assumed form y_o and angular velocity ω_o, will differ from y_o but a first approximation to the critical angular velocity is given by

$$\omega=\omega_o\sqrt{\frac{(y_o)_{max.}}{(y_1)_{max.}}} \qquad (2)$$

where $(y_o)_{max.}$ and $(y_1)_{max.}$ are respectively the maximum ordinates of the assumed and derived forms.

If y_o and y_1 have the same shape, i.e. if $y_1=ay_o$, the result would be exact since centrifugal force is proportional to $\omega^2 y$ and at all points along the shaft y_1 will be greater or less than y_o in the ratio $\frac{\omega_o^2}{\omega^2}$ the derived deflection being equal to the initial assumption at the critical speed ω.

If the shape of y_1 is appreciably different from that of y_o, we can repeat the process using y_1 as the assumed deflected form.

CONVERGENCE OF THE STODOLA METHOD.

To examine the convergence of this process, we first note that the differential equation for the lateral vibration of a straight loaded beam in which lateral dimensions are sufficiently small for rotary inertia to be neglected, that is

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) = -\frac{w}{g}\,\frac{\partial^2 y}{\partial t^2}$$

becomes identical with equation (1) in the case of stationary or " normal " modes of vibration in which

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$

ω being here the " pulsatance ", i.e. $2\pi \times$ the frequency of vibration.

The functions $f_1, f_2 \ldots f_n, \ldots$ then correspond to the " normal functions " of a vibrating beam and hence possess the property that

$$\int_o^l w(x)\, f_m(x) f_n(x).\, dx = 0 \quad (m \neq n) \qquad (3)$$

where $x=o$, $x=l$ are the end points of the beam system.[2]

The initial assumption y_o in the Stodola method can therefore be expressed in terms of a series of "normal functions", that is

$$y_o = a_1 f_1(x) + a_2 f_2(x) \ldots + a_n f_n(x) \ldots \qquad (4)$$

where the coefficient a_n is obtained by multiplying both sides by $w(x)\, f_n(x)$ and integrating between the limits o and l, i.e.

$$\int_o^l w(x)\, f_n(x)\, y_o(x)\, dx = a_n \int_o^l w(x)\, f_n^2(x)\, dx \qquad (5)$$

all other terms on the right-hand side being zero on account of (3).

The component $a_1 f_1(x)$ will produce an equal deflection with angular velocity ω_1, and in general $a_n f_n(x)$ will produce deflection $a_n f_n(x)$ with angular velocity ω_n. Remembering that deflection is proportional to ω^2, the assumed angular velocity ω_o applied to the deflected form y_o will, from (4), produce a deflection

$$y_1 = a_1 f_1(x)\frac{\omega_o^2}{\omega_1^2} + a_2 f_2(x)\frac{\omega_o^2}{\omega_2^2} \ldots + a_n f_n(x)\frac{\omega_o^2}{\omega_n^2} \ldots$$

$$= \frac{\omega_o^2}{\omega_1^2}\left\{a_1 f_1(x) + a_2 f_2(x)\left(\frac{\omega_1^2}{\omega_2^2}\right) \ldots + a_n f_n(x)\left(\frac{\omega_1^2}{\omega_n^2}\right) \ldots\right\} \qquad (6)$$

Since $\omega_1 < \omega_2 \ldots < \omega_n$, the coefficient of the term $a_1 f_1(x)$ in (6) is greater compared with the coefficients of the other terms than is the case in (4) and by repetition of the Stodola process, using (6) as the initial deflected form, we obtain ultimately a derived form in which all terms are negligible compared with that containing $a_1 f_1(x)$, while application of equation (2) will give $\omega = \omega_1$. Thus the Stodola method converges to give the deflected form and angular velocity at the first or lowest initial speed.

In more general terms, the Stodola method is an iteration process which converges to give the solution of differential equation (1) corresponding to the lowest Eigen value ω_1.

This property is of great importance in the investigation of critical speeds of rigidly-coupled rotors, as will be apparent later.

THEORY OF CONTINUOUS BEAMS WITH NON-UNIFORM CROSS-SECTION.

In the application of the method to coupled rotor systems, the problem of calculating the bending moment and deflection due to a given loading is much more difficult than in the case of a simple two-bearing shaft since the rotors form a continuous beam system and the cross-section is generally non-uniform. A graphical method of estimating the statically indeterminate support moments has been given by Stodola[3] and by E. S. Andrews[4], but the writer has found that a direct application of Clapeyron's

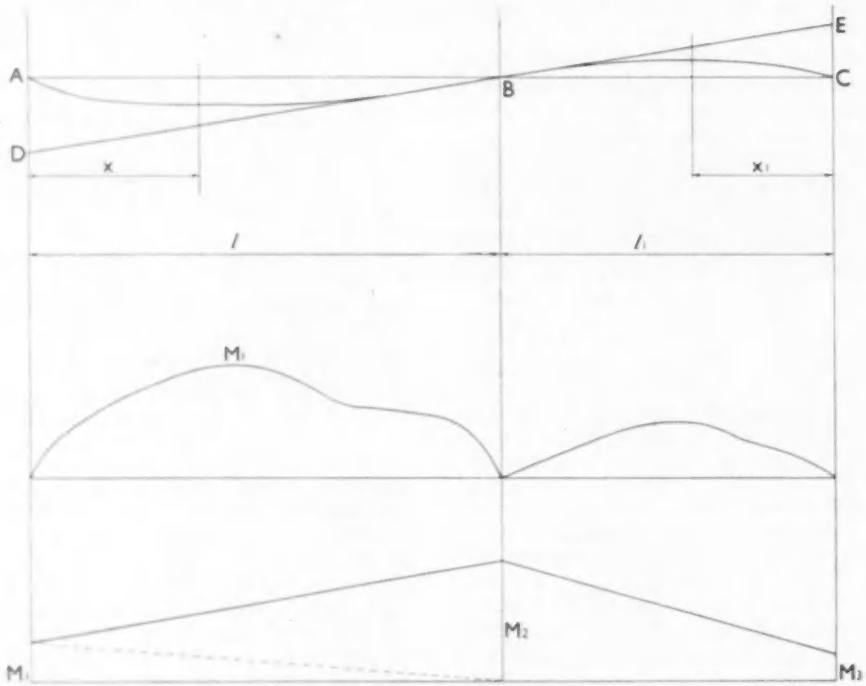

Fig. 2.—A theorem of three moments.

"Theorem of Three Moments" in a form modified to take into account the variation of cross-section is very simple and convenient.

Referring to fig. 2, AB, BC are two adjacent spans and DE is the tangent to the elastic line at B. Then by similar triangles.

$$\frac{AD}{AB} = \frac{EC}{BC} \qquad (7)$$

while by the second theorem of Mohr

$$AD = \int_o^l \frac{Mx}{EI}\, dx \qquad (8)$$

and $$EC = -\int_o^{l_1} \frac{Mx_1}{EI}\, dx_1 \qquad (9)$$

Now $AB=l$, $BC=l_1$, and substituting for AD and EC from (8) and (9) in (7) we get

$$\frac{1}{l}\int_o^l \frac{Mx}{EI}\, dx + \frac{1}{l_1}\int_o^{l_1} \frac{Mx_1}{EI}\, dx_1 = o \qquad (10)$$

To facilitate the evaluation of the integrals we express (10) in the form

$$\frac{1}{EI_o}\cdot\frac{1}{l}\int_o^l M\left(\frac{I_o}{I}\right)x.dx + \frac{1}{EI_o}\cdot\frac{1}{l_1}\int_o^{l_1} M\left(\frac{I_o}{I}\right)x_1.dx_1 = o \qquad (11)$$

where I_o is a suitable constant value of inertia (say, approximately the mean value for the beam system). Then since E is constant, (11) becomes

$$\frac{1}{l}\int_o^l M\left(\frac{I_o}{I}\right)x.dx + \frac{1}{l_1}\int_o^{l_1} M\left(\frac{I_o}{I}\right)x_1\, dx_1 = o \qquad (12)$$

Considering for the present the span AB, the bending moment M is the algebraic sum of the "free" bending moment M_f (i.e. the bending moment produced by the given loading on a span freely supported at A and B) and the fixing moments given by the ordinates of a straight line joining the support moments M_1 and M_2 due to the attachment of AB to adjacent spans.

Thus $$M = M_f + M_1\frac{l-x}{l} + M_2\frac{x}{l}$$

and $\int_o^l M\left(\frac{I_o}{I}\right) x.\, dx$ can be expressed as

$$\int_o^l M_f\left(\frac{I_o}{I}\right)x.dx + \int_o^l M_1\left(\frac{l-x}{l}\right)\left(\frac{I_o}{I}\right)x.dx + \int_o^l M_2\frac{x}{l}\left(\frac{I_o}{I}\right)x.dx$$

with a corresponding result for

$$\int_o^{l_1} M\left(\frac{I_o}{I}\right)\, x_1.dx_1.$$

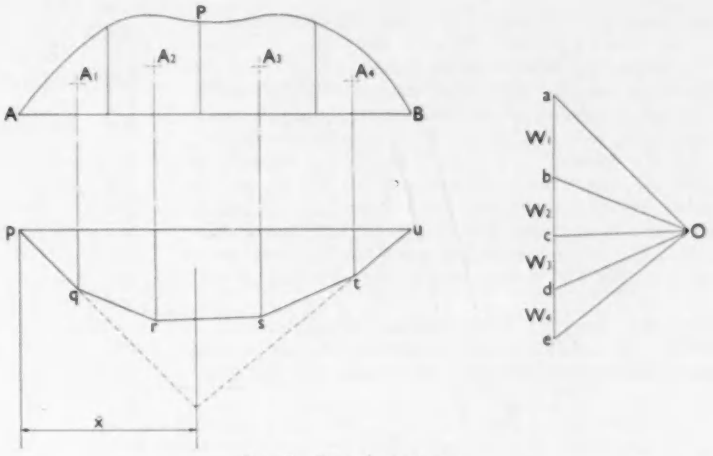

Fig. 3.—Funicular polygon.

Using these results, equation (12) becomes

$$\frac{1}{l}\int_o^l M_f\left(\frac{I_o}{I}\right)x.dx + \frac{1}{l}\int_o^l M_1\left(\frac{l-x}{l}\right)\left(\frac{I_o}{I}\right)x.dx + \frac{1}{l}\int_o^l M_2\frac{x}{l}\left(\frac{I_o}{I}\right)x.dx + \frac{1}{l_1}\int_o^{l_1} M_f\left(\frac{I_o}{I}\right)x_1\, dx_1 + \frac{1}{l_1}\int_o^{l_1} M_2\frac{x_1}{l_1}\left(\frac{I_o}{I}\right)x_1 dx_1 + \frac{1}{l_1}\int_o^l M_3\left(\frac{l_1-x_1}{l_1}\right)\left(\frac{I_o}{I}\right)x_1.dx_1 = 0 \qquad (13)$$

Equation (13) is a general form of Clapeyron's Theorem of Three Moments for beams of non-uniform cross-section and appears rather complicated but the values of all the integrals can be obtained in a very simple manner. We note that

$$\int_o^l M_f\left(\frac{I_o}{I}\right)x.dx$$

is the first moment about A of the "modified" free bending moment diagram for span AB, the "modified" diagram being the bending moment diagram with ordinates at all points multiplied by the factor $\left(\frac{I_o}{I}\right)$. Similarly $\int_o^l M_1\left(\frac{l-x}{l}\right)\left(\frac{I_o}{I}\right)x.dx$ is the first moment about A of the "modified" fixing moment diagram due to support moment M_1. The method of evaluation of the other integrals in (13) in this manner will be obvious, the first moments for the last three being about C.

Since the deflection diagram will be derived from the bending moment diagram by the funicular polygon construction, it is very convenient to use one of the properties of this construction to calculate the above first moments of the bending moment diagrams.

Referring to fig. 3, the bending moment diagram APB is divided into areas A_1, A_2, A_3, A_4 by vertical

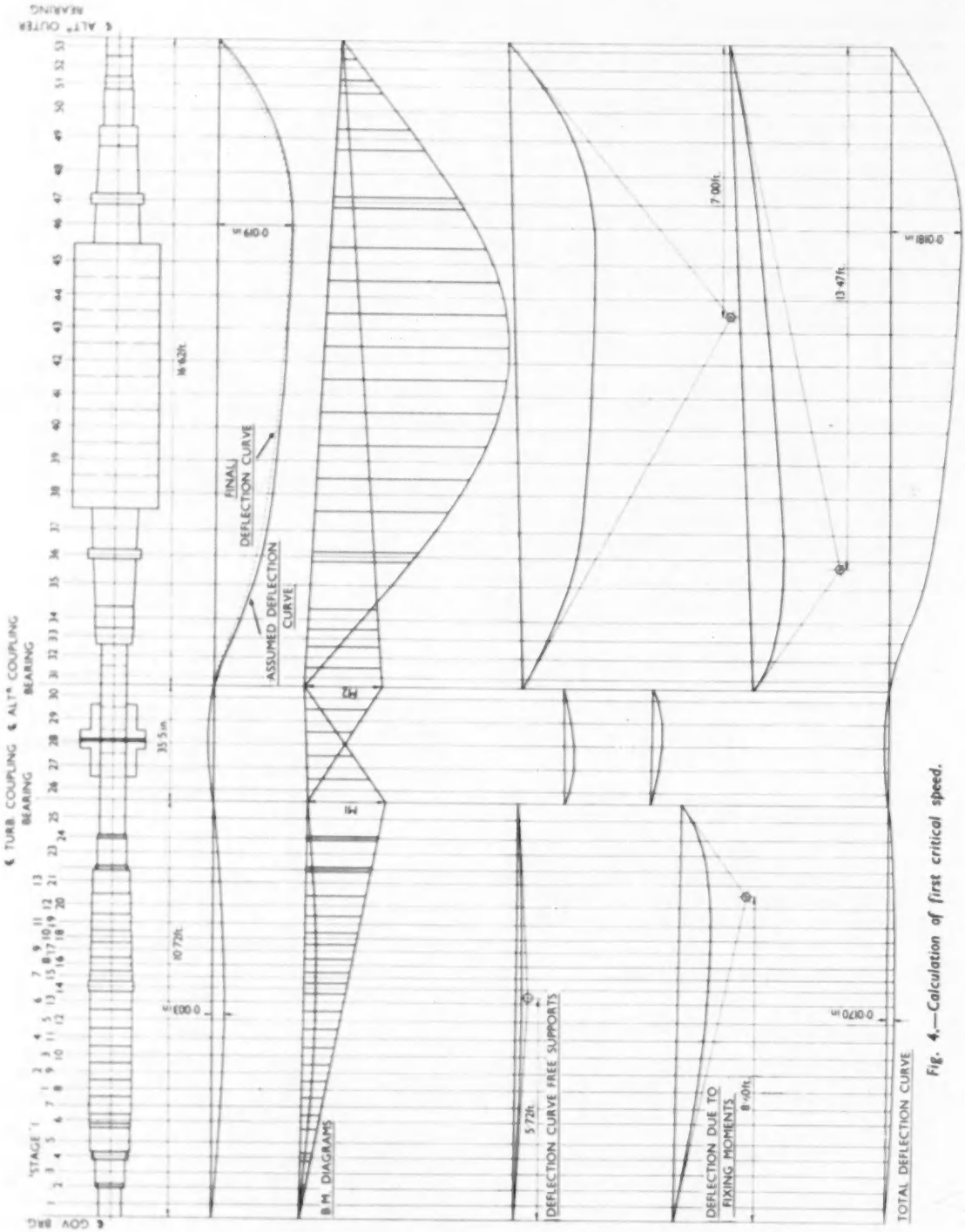

Fig. 4.—Calculation of first critical speed.

lines and over the part of the span covered by each area the moment of inertia of the cross-section of the shaft is constant, the values being respectively I_1, I_2, I_3, I_4. Each area is multiplied by the non-dimensional factor $\left(\frac{I_o}{I}\right)$ to give the "loads" $W_1 \ldots W_4$. The rays *oa*, *ob*, etc. are then drawn in the normal manner and set down between the verticals through the centroids of the areas $A_1 \ldots A_4$ to form the "bending moment diagram" produced by "loading", $W_1 \ldots W_4$, *pq* being parallel to *oa* and so forth. By the well-known properties of the funicular polygon, the vertical through the intersection of *pq* and *ut* produced is the line of action of a system of forces proportional to W_1, W_2...etc. i.e., this vertical passes through the centroid of the modified bending moment diagram.

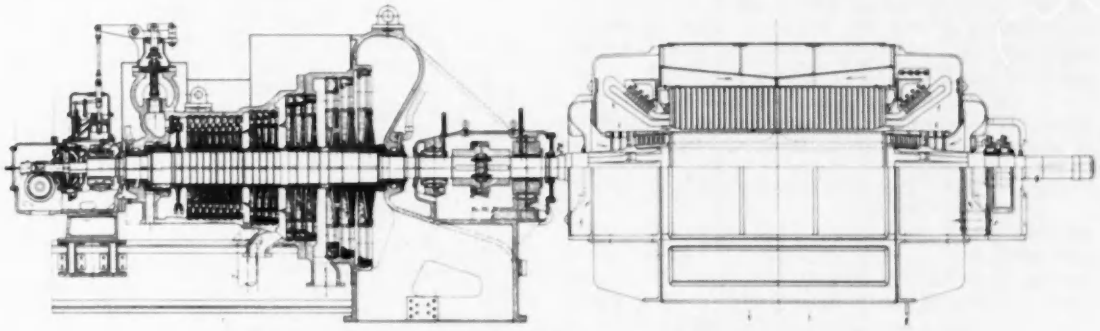

Fig. 5.—Cross-section of 10 MW turbo-generator for Bloemfontein power station.

Thus the first moment of the modified bending moment diagram about A is

$$\sum(W)\times\bar{x}$$

$$\text{i.e. } \left(A_1\frac{I_o}{I_1}+A_2\frac{I_o}{I_2}+A_3\frac{I_o}{I_3}+A_4\frac{I_o}{I_4}\right)\times\bar{x} \qquad (14)$$

NUMERICAL EXAMPLE.

The technique of application will now be explained by working through the main features of a typical calculation.

Fig. 5 shows the cross-section of a single-casing, 3,000 r.p.m. turbo-generator of 10 MW rating, the turbine and generator rotors being rigidly coupled to form a four-bearing continuous system. The first critical speed of the turbine rotor uncoupled is about 1,920 r.p.m. and that of the generator about 1,270 r.p.m. At the first critical speed of the coupled system the generator rotor will dominate and the amplitude of the turbine rotor will be relatively small.

The relevant features of the rotor system are shown at the top of fig. 4 and the first step in the calculation is to sketch freehand the first assumption for the deflected form of the coupled rotors at the lowest critical speed. We assume $\frac{\omega^2}{g}=50\ \text{in}^{-1}$ corresponding to 1,327 r.p.m. and a mean inertia $I_o=1{,}200\ \text{in}^4$. Following the normal technique of the graphical method, the rotors are divided up into sections, each having constant shaft diameter, and the centrifugal loading is calculated. The free bending moment curves for the turbine and generator rotors are then drawn by the usual funicular polygon construction. The centrifugal loading on the short middle span is very small and has been omitted from this calculation but its inclusion would not present any special difficulty.

The statically indeterminate support moments are M_1 and M_2 at the bearings on either side of the coupling, the outer ends being freely supported if we neglect the effect of the overhanging ends. We draw the fixing moment diagrams for all spans as shown on fig. 4, using arbitrary ordinates of two inches for M_1 and M_2.

The vertical lines used for subdividing the rotor spans are used to divide the bending moment diagrams into suitably small areas, for each of which the area is multiplied by $\left(\frac{I_o}{I}\right)$ and used as an increment of "load". Further application of the funicular polygon method gives the "bending moment" diagrams, the ordinates of which, after division by EI_o, give the deflection diagrams. The polar distances of the funicular polygon are calculated so that the scale of the derived deflection diagrams is the same as that of the initially assumed curve.

The horizontal positions of the centroids of the modified bending moment diagrams are found by the intersection of the outermost rays of the funicular polygon, and the first moments of each diagram about the relevant supports are found as in equation (14).

The shaft diameter over the span carrying the coupling is very nearly constant, so that the modified

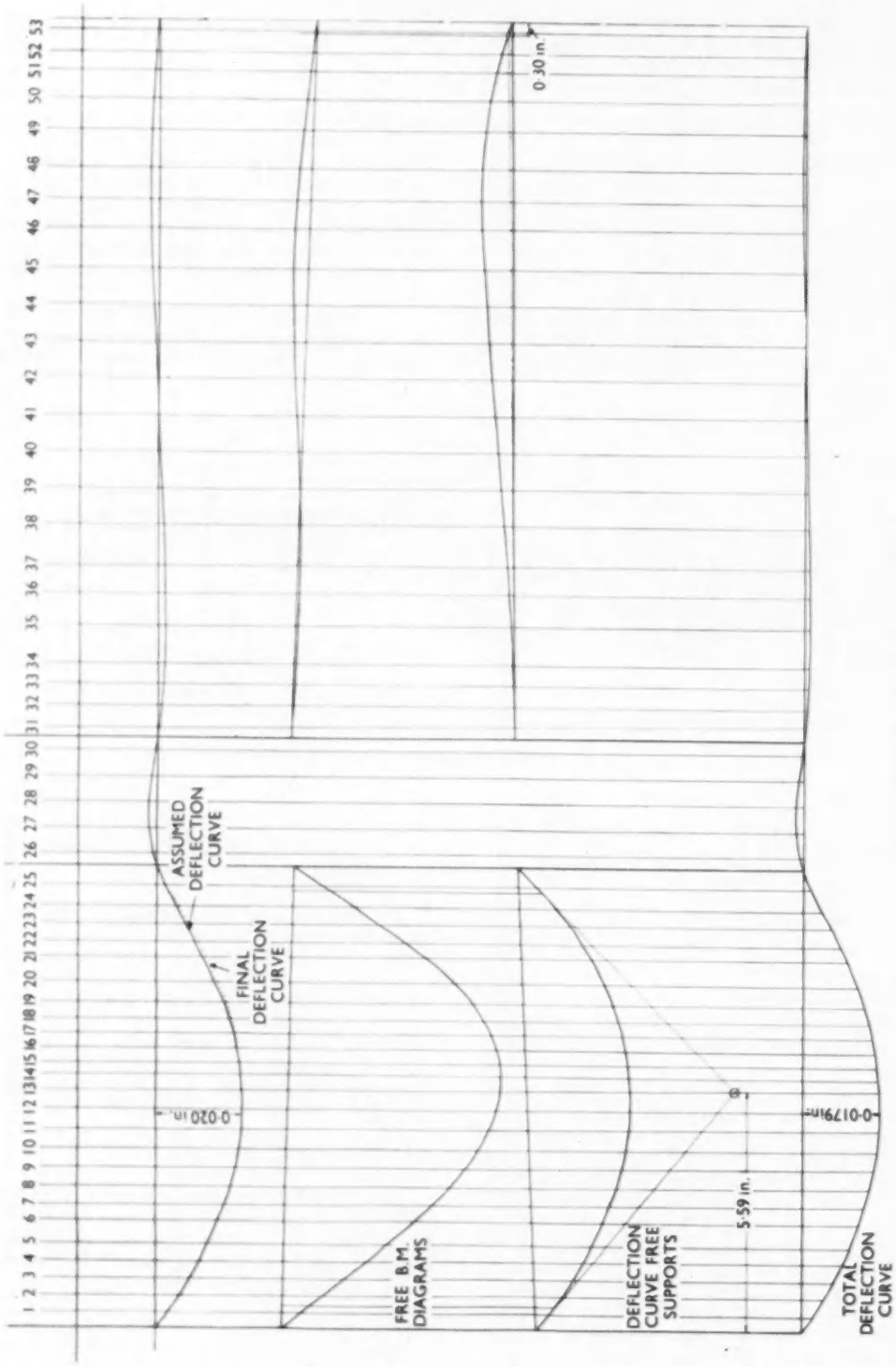

Fig. 6.—Calculation of second critical speed.

bending moment diagrams due to the support moments remain as triangles, for which the positions of the centroids are known.

From the usual tabulation of loads, etc., used in the funicular polygon construction, the areas of the various modified bending moment diagrams, i.e. the values of

$$\sum M\left(\frac{I_o}{I}\right).\Delta x$$

are :

L.H. span.	Free B.M. diagram, 2·801 in.^{2}
	Fixing moment diagram (M_1=2 in.), 24·623 in.^{2}
Central span.	Fixing moment diagram (M_1=2 in.), 17·650 in.
	Fixing moment diagram (M_2=2 in.), 17·650 in.
R.H. span.	Free B.M. diagram, 35·760 in.
	Fixing moment diagram (M_2=2 in.), 23·847 in.

The areas of the fixing moment diagrams are of course proportional to M_1 and M_2, so that if the support moment at the bearing on the left of the coupling is of undetermined magnitude M_1 (reckoned in inches on the diagram), the value of, say, the area of the fixing moment diagram on the left-hand span will be

$$\left(\frac{M_1}{2}\right)\times 24{\cdot}623 \text{ in.}^2$$

We now apply the theorem of three moments, in the form of equation (13) to each of the pairs of adjacent spans in turn, the numerical values of the various integrals being written down immediately from the above table and the centroid co-ordinates on fig. 4.

Thus for the left-hand pair of spans, equation (13) gives

$$2{\cdot}801\times\frac{5{\cdot}72}{10{\cdot}72}+\left(\frac{M_1}{2}\right)\times 24{\cdot}623\times\frac{8{\cdot}40}{10{\cdot}72}+\left(\frac{M_1}{2}\right)\times 17{\cdot}650\times\frac{2}{3}+\left(\frac{M_2}{2}\right)\times 17{\cdot}650\times\frac{1}{3}=0$$

and for the right-hand pair

$$\left(\frac{M_1}{2}\right)\times 17{\cdot}650\times\frac{1}{3}+\left(\frac{M_2}{2}\right)\times 17{\cdot}650\times\frac{2}{3}+\left(\frac{M_2}{2}\right)\times 23{\cdot}847\times\frac{13{\cdot}47}{16{\cdot}62}+35{\cdot}760\times\frac{7{\cdot}00}{16{\cdot}62}=0$$

Solving these equations simultaneously for M_1 and M_2 we get

$$M_1=0{\cdot}091 \text{ in.} \qquad M_2=-0{\cdot}986 \text{ in.}$$

The ordinates of the deflection diagrams for the assumed support moments of two inches are multiplied by the appropriate ratios $\left(\frac{M_1}{2}\right)$ and $\left(\frac{M_2}{2}\right)$ and then added algebraically to the deflection diagram for the "free" bending moment diagram to give the final deflection curve shown on fig. 4.

Since the generator rotor dominates here, the corrected value of the first critical speed is obtained from the ratios of the maximum ordinates of the curves for the generator rotor and equation (2) then gives :

First critical speed=

$$1327\sqrt{\frac{0{\cdot}019}{0{\cdot}0181}}=1360 \text{ r.p.m.}$$

The ordinates of the derived curve are multiplied by $\frac{0{\cdot}019}{0{\cdot}0181}$ and superimposed on the initially assumed curve in fig. 4.

In this instance the similarity is very close, the initial guess for the deflected form having been good. If the initial and derived curves were appreciably different, the latter would be used as the initial assumption for a second calculation and the convergence of the Stodola method in the case of the first critical speed would ensure a more accurate result.

At the second critical speed of the coupled system the turbine rotor will dominate and the ordinates of the deflected form of the generator rotor will be relatively small. It must also have a positive and negative loop, for with a single loop the centrifugal forces would be greater than the elastic restoring forces at the second critical speed.

Referring to fig. 6, the calculation for the second critical speed is carried out in the same manner as for the first, the assumed value of $\frac{\omega^2}{g}$ being 125 in^{-1}, corresponding to 2,097 r.p.m. The fixing moment diagrams do not have to be repeated since the data relevant to them for insertion in the terms of the theorem of three moments is already available from the first critical speed calculations.

The ratio of the maximum ordinates of the turbine rotor is used to calculate the corrected value of critical speed, since this rotor dominates and equation (2) gives :

Second critical speed=$2{,}097\sqrt{\frac{0{\cdot}02}{0{\cdot}0179}}=2{,}217$ r.p.m.

DIFFICULTIES ASSOCIATED WITH HIGHER CRITICAL SPEEDS.

The principal difficulty in the calculation of the second or higher critical speeds is to make a sufficiently accurate initial assumption for the deflected form. If the assumed form is not very nearly correct, the first derived curve will be less correct and repetition of the process using the derived curve as the initial assumption will give results further and further from the truth until ultimately we end up with the deflected form for the first critical speed, to which, as we have shown, the Stodola process converges.

With experience, it is possible to observe the departure of the first derived curve from the initial assumption and make an independent guided second guess, but the process of adjustment is usually time-consuming and laborious.

A more direct method is as follows :

In the proof of the convergence of the Stodola method, it is shown how the component of the initial assumption y_o containing the normal function $f_1(x)$

becomes dominant. If, however, this function were absent from y_o, it is readily seen that the normal function $f_2(x)$ would become dominant, while if both $f_1(x)$ and $f_2(x)$ were absent, $f_3(x)$ would become dominant and so forth.

In the case of the second critical speed, y_o can be freed from $f_1(x)$ by using the results of the calculation of the first critical speed and the orthogonal property of the normal functions defined by equation (3).

Let the initial assumption for the second critical speed be y_o which will, in general, contain a component $a_1 f_1(x)$ that is

$$y_o = a_1 f_1(x) + a_2 f_2(x) \ldots + a_n f_n(x) \ldots$$

Multiplying both sides by $w(x)\, f_1(x)$ and integrating over the whole length of the beam system, we get

$$\int_o^l w(x) f_1(x) y_o . dx = a_1 \int_o^l w(x) f_1^2(x) . dx + a_2 \int_o^l w(x) f_1(x) f_2(x) dx \ldots$$

but by equation (3) all the terms on the right-hand side except the first vanish.

Then
$$\int_o^l w(x) f_1(x) y_o . dx = a_1 \int_o^l w(x) f_1^2(x)\, dx \quad (15)$$

A good approximation to $f_1(x)$ is known from the calculation of the first critical speed so that the integrals in equation (15) can be evaluated approximately by step by step summation over the beam system, giving

$$a_1 = \frac{\Sigma w(x) f_1(x) y_o . \Delta x}{\Sigma w(x) f_1^2(x) . \Delta x} \quad (16)$$

Thus from our assumed deflection y_o, for the second critical speed we subtract $a_1 \times$ the normal function derived from the first critical speed calculation and obtain a curve substantially free from $f_1(x)$. The use of this curve in the Stodola process will result in convergence to the normal function of the second critical speed. The number of repetitions is, however, limited since any remaining trace of $f_1(x)$ would ultimately become dominant.

For the third critical speed, $f_1(x)$ and $f_2(x)$ can be eliminated from y_o in a similar manner, but the tendency for remaining traces of $f_1(x)$ to become dominant becomes more marked in the application to the third and higher critical speeds.

SIMPLIFIED METHOD FOR HIGHER CRITICAL SPEEDS OF COUPLED SYSTEMS.

From studies of a number of calculations of typical rotor systems in the Turbine Design Department at Erith, it became apparent that when one rotor dominates, the centrifugal loading on the other rotors is small and does not have much bearing on the result. It can therefore be ignored and the system calculated as if all rotors except the one dominant at the particular critical speed were weightless.

This is particularly useful for critical speeds higher than the first since the higher critical speed becomes the first critical speed of the modified system and the Stodola process converges. Furthermore, we need guess the initial deflected form only for the dominant rotor.

Applying this technique to the present example, for the first critical speed (generator rotor dominant) we omit the term

$$\frac{1}{l}\int_o^l M_f \left(\frac{I_o}{I}\right) x .\ dx$$

for the turbine rotor from the theorem of three moments for the left-hand pair of spans. The values of the support moments then become :

$$M_1 = 0{\cdot}190 \text{ in.}, \quad M_2 = -1{\cdot}005 \text{ in.}$$

The maximum ordinate of the derived deflection curve for the generator rotor then becomes 0·018 in. instead of 0·0181 in., the effect on the value of the critical speed being negligible. The effect of neglecting the centrifugal loading on the generator rotor in the case of the second critical speed is even smaller.

CONCLUSION.

The methods outlined in this article enable the critical speeds of rigidly-coupled rotor systems to be estimated with high accuracy, but in the case of critical speeds higher than the first, the amount of calculation is rather large, even though it is of a fairly simple nature.

In very many cases, however, each rotor becomes dominant in turn and the suggested process of ignoring the centrifugal forces on all rotors other than the dominant one reduces the problem to one of determining the first critical speed of a modified system. The Stodola method is then convergent, and the amount of calculation required is markedly less than that normally involved in deriving a reasonably close approximation to the mode of a second or higher critical speed.

This simple method cannot be used in cases where a dominant rotor has a multiple loop, having already been dominant with a single loop at a lower critical speed, but this state of affairs could scarcely arise on any normal turbo-machine except at rotational speeds far in excess of those at which it could run.

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New Plant for Super-tension Cable Production

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Works Manager and Chief Engineer, Pirelli-General Cable Works Ltd.

THE highest-rated oil-filled cable to be made in the United Kingdom is the 301 kV cable supplied to the Aluminum Company of Canada for its Kemano-Kitimat project. This cable* was designed, manufactured and installed by Pirelli-General Cable Works Ltd., in conjunction with Pirelli of Milan, and repeat orders for further lengths of cable have since been received.

The successful development of the Kemano-Kitimat cable entailed the setting of new and higher standards of manufacture and testing, and the equipment and techniques evolved have now become part of the normal production process for Pirelli-General super-tension cables at the Eastleigh Works. A specially designed paper-lapping machine has been installed, capable of applying simultaneously up to 200 layers of paper insulation under precisely controlled conditions of tension and registration. The machine is housed in a fully air-conditioned enclosure, so that the entire lapping operation can be carried out in a controlled atmosphere.

This equipment, the first of its kind to be used on full-scale production in this country, enables a consistently high standard of electrical insulation to be maintained automatically, irrespective of external atmospheric conditions during manufacture.

New development and test bays associated with the electrical laboratory have been equipped with the latest apparatus for carrying out development work and type approval tests on cables for the highest voltages.

To-day there are over 600 feeder-miles of oil-filled cable installed or in process of installation in this country, and it is estimated that there are nearly 2,000 feeder-miles installed throughout the world. The development and progress of oil-filled cable practice since the first experimental 132 kV installation by Pirelli S.p.A., in Italy in 1924, has been the subject of a previous article in this journal.†

PAPER-LAPPING PROBLEMS.

Until recently, the maximum permissible working stress with oil-filled cables was 90 kV cm. In designing 301 kV cable, however, it was realised that if the cable dimensions and the cost of manufacture were to be kept within reasonable limits, the insulation would have to withstand even higher stresses. A maximum working stress of 120 kV cm. was considered feasible, provided that a dielectric of nearly perfect quality could be ensured.

One of the chief difficulties in the way of achieving this standard is the presence of wrinkles and creases

Fig. 1.—Section of 301 kV oil-filled cable.

* *G.E.C. Journal*, Vol. 22, No. 3, pp. 166-171, July, 1955.
† *G.E.C. Journal*, Vol. 21, No. 4, pp. 199-214, October, 1954.

in the paper insulation. Such defects reduce the impulse strength, which, with present-day specifications, is the limiting factor in high-voltage cable design.

Wrinkles and creases can occur during manufacture when the cable is being taken round the haul-off wheel of the paper lapping machine or when it is being wound on the drum ready for drying. They are caused by the inability of the layers of paper tape to slip one on the other when the cable is bent, with the result that the papers on the inside of the bend buckle in the spaces between the tapes of the outer layers.

The difficulty can be avoided by very careful control of the tension with which the papers are applied, and by accurate grading of these tensions radially through the insulation. In order to do this, however, the moisture content of the paper at the time of lapping must be known, so that due allowance can be made for the shrinkage which occurs during drying.

Fig. 3.—Slitting the reel of paper into pads for use in the machine.

NEW PAPER-LAPPING MACHINE.

To maintain complete control of the paper-lapping operation, Pirelli-General have installed a precision lapping machine designed by Pirelli S.p.A. of Milan, and manufactured for them in Milan. The machine consists of 20 reversible lapping heads, each capable of applying 10 papers, so that up to 200 layers of insulation can be applied simultaneously. It is thus

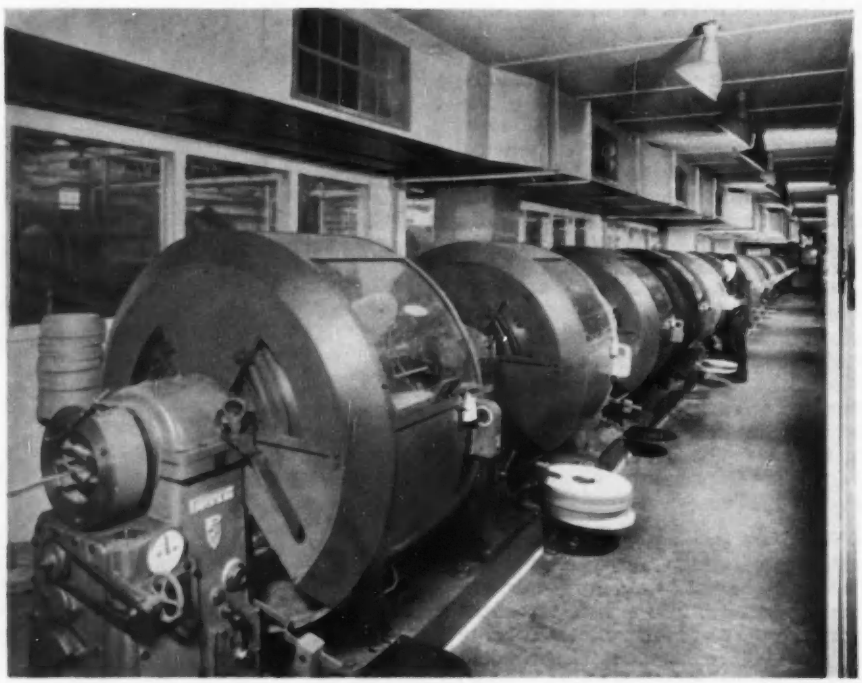

Fig. 2.—A view along the 150 ft. length of the new Pirelli paper-lapping machine.

possible to insulate cables for 300 kV and above in one operation. A section of 301 kV oil-filled cable, showing the paper insulation, is illustrated in fig. 1.

The lapping machine (fig. 2) is contained in a fully air-conditioned chamber having a volume of 40,000 cu. ft. Also in the enclosure are the paper-cutting machine (fig. 3) and storage racks for the cut paper pads so that the paper is properly conditioned before use. The entire lapping process can thus be carried out under constant controlled conditions of temperature and humidity.

The most important feature of the paper-lapping machine is the paper tensioning device. Tension is imparted to the paper by means of a spring-controlled band brake acting on each pad holder. Fig. 5 shows the pads in position in one of the lapping heads. The paper passes to the cable over four rollers, one of which is attached to a pivoted lever, which is itself spring-controlled and coupled by link work to the band brake. The paper tension is predetermined by selecting springs (which are interchangeable) of the requisite strength. Valuable properties of the paper tensioning mechanism are that it is unaffected by centrifugal forces and that it maintains correct tension even when the machine is stopped and despite the slight, unavoidable backlash.

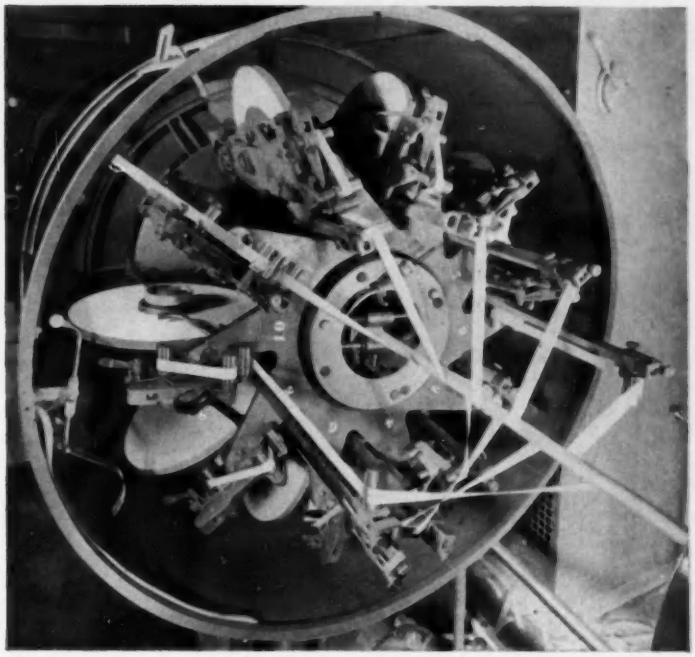

Fig. 5.—One of the lapping heads, showing paper tapes being applied to a 301 kV cable.

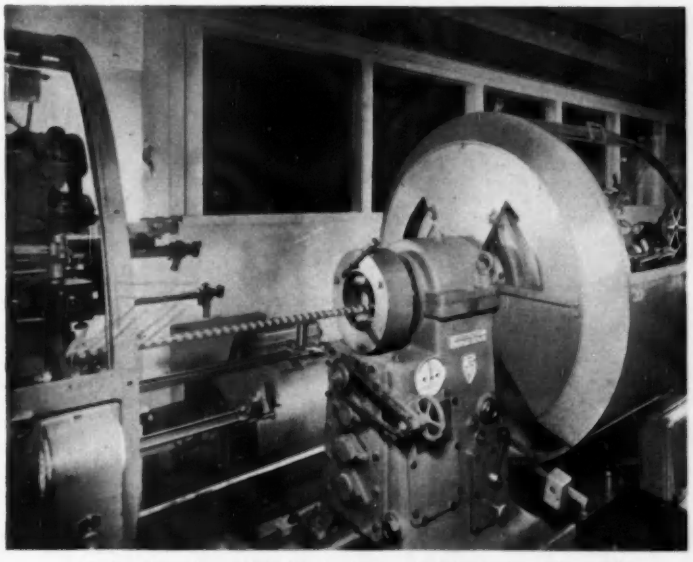

Fig. 4.—View of one of the heads showing direction and speed controls.

Accurate registration of the paper tapes is controlled by means of a fine graduated screw adjustment to the position of the last roller over which each paper passes. There is also an adjustment for the angle of the paper holders to ensure that the centre plane of the paper pad has the correct lead angle to the cable. The adjustment moves all 10 holders of a head together.

Each of the 20 lapping heads is driven through a positive infinitely-variable speed gear, the direction of rotation being changed when required simply by the movement of a small lever (fig. 4). The machine can be stopped or started by push button controls situated on each head. Simultaneous braking of all heads is provided by an electro-hydraulic mechanism operating through a layshaft.

For safety in operation, all the lapping heads are partially enclosed by a transparent guard ; a knock-off device on each head stops the machine if a paper breaks or runs

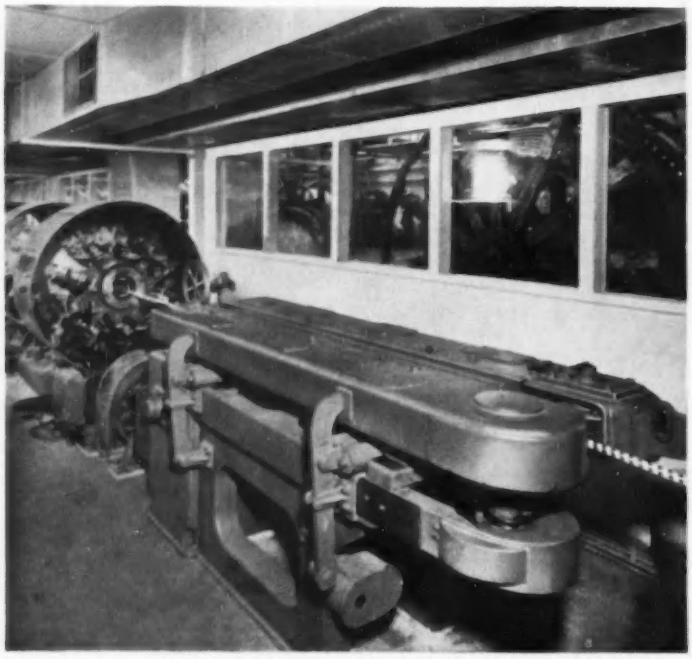

Fig. 6.—Final head of machine and caterpillar haul-off, with length of 33 kV cable passing through.

out or if a guard is lifted, and also lights a red lamp on the head affected. Cable is drawn through the machine by means of a caterpillar type haul-off (fig. 6).

AIR-CONDITIONING PLANT.

An air duct runs the full length of the paper-laying enclosure, and conditioned air is blown from a number of outlet louvres across the top of the enclosure. After circulating, the air is extracted through apertures at floor level by a fan in the return air ducting, and is returned to the conditioning plant with a proportion of fresh outside air, which is continuously added.

To maintain constant humidity, the conditioning plant (fig. 7) incorporates refrigeration units providing chilled water, which is pumped to sprays in the washer and returned to the refrigeration unit for re-chilling. In the washer, the air becomes saturated with water vapour at the low temperature. When it is subsequently warmed in the reheaters to the required temperature its relative humidity is reduced to the predetermined value.

Once the required temperature and humidity values have been set the whole plant is automatically controlled to maintain even conditions through summer and winter. Over a measured test period, the maximum and minimum outside temperature and relative humidity conditions varied between 34 deg. and 63 deg. F., with relative humidities of 48 per cent. to 84 per cent. Throughout this period conditions in the enclosure remained constant without attention at the preset values of 70 deg. F. and 40 per cent. relative humidity.

The lapping machine take-off stands are partitioned off from the main enclosure so that drums can be removed and replaced without opening the main enclosure to the outside atmosphere.

PAPER QUALITY.

The quality of the wood pulp paper used for lapping is most important and factors such as thickness, porosity and tensile strength are very carefully tested by the cable maker. Of particular significance is the moisture content, as the correct allowance must be made for shrinkage during the drying process. All paper is hygroscopic, and in its natural state can contain between 4 and 10 per cent. of moisture, representing anything from 9 to 22 gallons of water per ton of paper.

To obviate the effect of these variations the paper bales are cut into pads and stored in racks in the enclosure for about seven days before use to ensure the correct moisture content.

ELECTRICAL LABORATORY.

For the purpose of carrying out development and type approval tests on super-tension cables, the

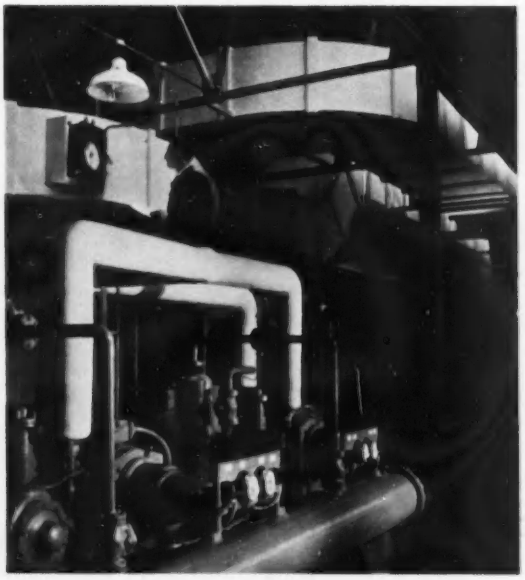

Fig. 7.—Air-conditioning equipment and ducting.

previous test facilities were moved to a new site and new equipment added (fig. 8, 10 and 11). In the development bay, a Micafil impulse generator has been installed having a stored energy of 38,500 watt seconds and an open circuit voltage of 1,600 kV peak. The generator is capable of applying impulses up to 1,400 kV peak on samples of cables, including accessories, up to about 25 yards in length. A feature of the impulse generator, which is of Swiss design and manufacture, is its small size in relation to its output. Its design is somewhat unusual in that all the capacitors are contained in a central column of bakelised paper, instead of in several porcelain columns. The charging unit consists of a mechanical high-voltage rectifier operating at 200 kV D.C.

The impulse generator is remotely controlled from a desk in an elevated control room (fig. 9) overlooking the testing bay. Also in the control room is a cathode ray oscillograph for monitoring and photographing the applied impulses. The control room and the entire floor of the test enclosure are screened by a wire mesh counterpoise which, with all measuring-circuit screens, is earthed at a single point close to the impulse generator. The generator can be moved

Fig. 9.—Controlling and observing an impulse test on a 275 kV cable sealing end.

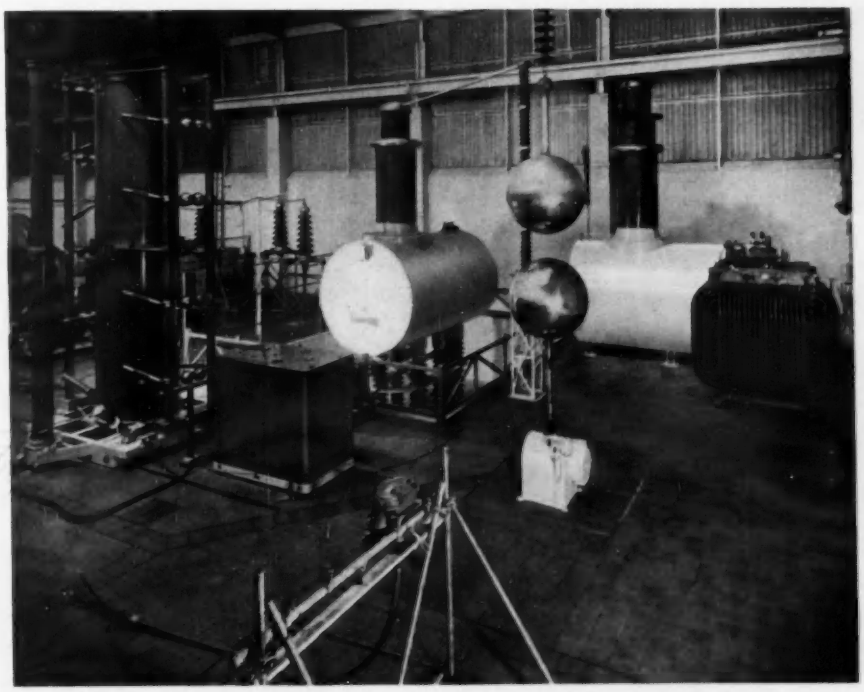

Fig. 8.—Newly-equipped development bay, with transformers and movable impulse generator.

on rails and is provided with four operating positions, each with a separate earthing electrode.

Also in the development bay are two 330 kV transformers which can be operated either in parallel or in cascade to give a total voltage of 660 kV to earth. Impulse and A.C. voltages are measured by a sphere gap between two 1-metre diameter copper spheres (fig. 8).

For routine testing of the long lengths of high-voltage cable which are now demanded, a further pair of transformers, each having a maximum voltage of 200 kV, has been installed in a second enclosure. These have a combined rating of 1,600 kVA continuously, or 2,800 kVA for half an hour; the transformers can be operated either in parallel, at 200 kV, or in cascade, at 400 kV (fig. 11).

Both transformers are provided with built-in variable reactors connected across the primary windings. The reactors are remotely controlled so that they can be adjusted to compensate for the capacity current of the load ; this results in the control equipment and the load on the supply being reduced to the minimum. These transformers are controlled from another desk in the main control room in which there is also a Schering type power factor measuring bridge, using an electronic detector in place of the more usual vibration galvanometer.

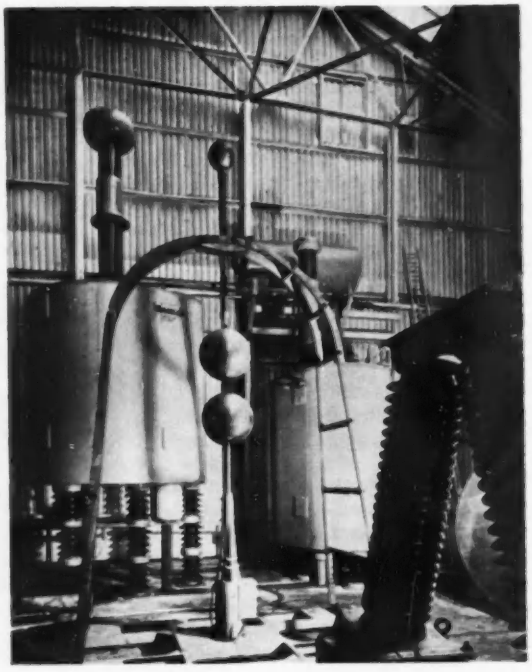

Fig. 11.—200 kV transformers and sphere gap for routine tests.

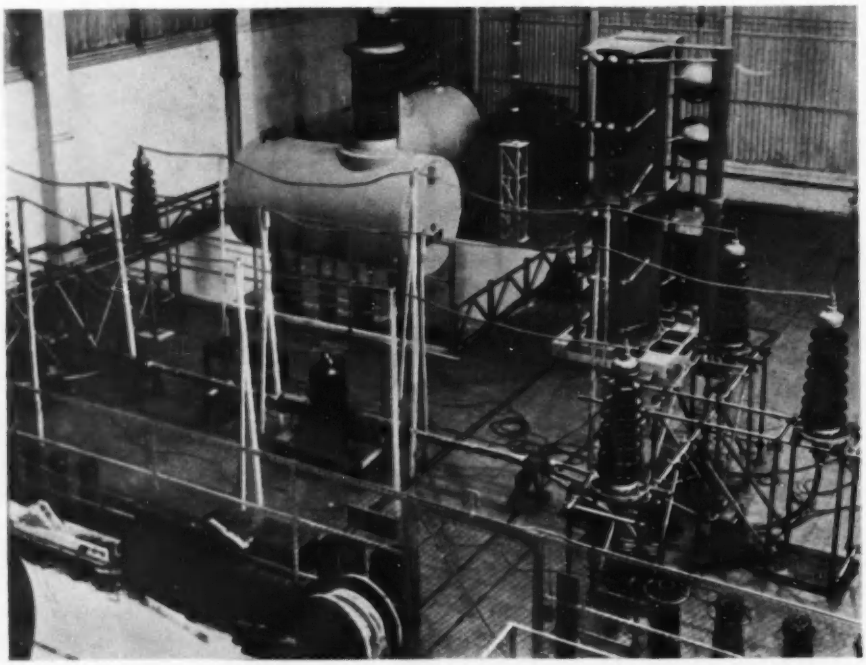

Fig. 10.—Length of 132 kV, 3-core cable, including joint and sealing ends, ready for test.

Design of Cathode-ray Tubes for Television

By R. C. HART, M.Sc., A.INST.P.
Research Laboratories.

Four factors contributing to the improvement of tube performance are discussed, and methods of achieving the best results with the techniques at present available are examined. It is shown that the beam angle, or divergence, is one of the most important design parameters of the tube, and consideration is given to the choice of beam angle in magnetic focus and electrostatic focus tubes. The author then describes means of controlling the beam angle with the various types of electron gun in use.

INTRODUCTION.

SINCE the post-war resumption of television broadcasting, there has been a continuous improvement in the quality of the received picture. The chief criticisms of the picture in the early post-war sets were its low brightness and small size. The steady increase in tube diameter of about 1 in. a year and the improvement in picture brightness due to E.H.T. voltage rising by about 1 kV a year, represent a considerable effort on the part of design and development engineers. This continuous development has necessitated changes in the design of cathode-ray tubes as new circuits, components and valves have become available.

Robust construction to resist damage in transit, stability of operation, and freedom from open circuits and short circuits to other electrodes, are among the generally accepted requirements for a reliable cathode-ray tube. Another is a good high-voltage performance with absence of inter-electrode leakages and sudden flashovers inside or outside the tube.

The steady rise in the final anode voltage to about 16 kV has emphasised the problem of maintaining adequate insulation between closely-spaced electrodes. Breakdowns caused by loose particles in the tubes may often be eliminated by a suitable form of "spot-knocking" treatment. Field emission of a more continuous kind may be avoided by careful attention to the condition of the metal surfaces of low-voltage electrodes where the electric field strength is high and spacings of about two millimetres or less are employed.

Early post-war tubes commonly suffered from ion burn of the screen, caused by negative ions from the low-potential region of the beam bombarding the screen. This has been eliminated by the use of ion traps or by aluminising. Ion traps employ a small magnetic field to separate the electron beam from the ion beam, and allow the ion beam to be trapped in a convenient part of the assembly.

The two common types of gun with ion traps, shown in fig. 1, are the skew-gun and bent-gun. In the former, the ion beam is deflected by an electrostatic field while the electron beam is corrected by the field of the ion trap magnet. In the latter type, the separation is obtained by tilting the gun and bringing the electron beam coaxial with the tube neck by means of the ion trap magnet.

When non-aluminised tubes are operated at final anode voltages greater than 10 kV there is a danger of positive ion burn of the screen, caused by the

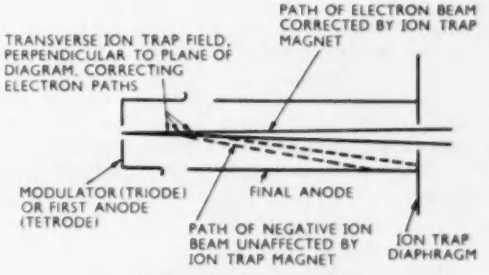
PATH OF ELECTRON BEAM CORRECTED BY ION TRAP MAGNET
TRANSVERSE ION TRAP FIELD, PERPENDICULAR TO PLANE OF DIAGRAM, CORRECTING ELECTRON PATHS
MODULATOR (TRIODE) OR FIRST ANODE (TETRODE)
FINAL ANODE
PATH OF NEGATIVE ION BEAM UNAFFECTED BY ION TRAP MAGNET
ION TRAP DIAPHRAGM

Fig. 1a.—Skew-gun ion trap.

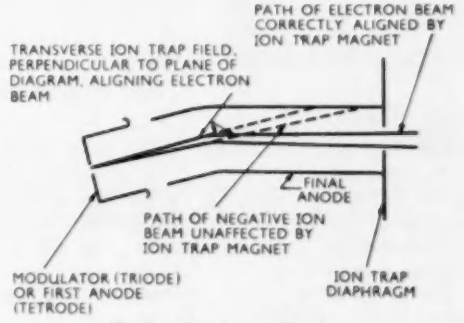
PATH OF ELECTRON BEAM CORRECTLY ALIGNED BY ION TRAP MAGNET
TRANSVERSE ION TRAP FIELD, PERPENDICULAR TO PLANE OF DIAGRAM, ALIGNING ELECTRON BEAM
FINAL ANODE
PATH OF NEGATIVE ION BEAM UNAFFECTED BY ION TRAP MAGNET
MODULATOR (TRIODE) OR FIRST ANODE (TETRODE)
ION TRAP DIAPHRAGM

Fig. 1b.—Bent-gun ion trap.

potential of the screen falling below that of the final anode. Aluminising completely avoids this difficulty. So long as the initial design is sound and the workmanship good, the screen quality will be satisfactory throughout the life of the tube. The end of life of the cathode-ray tube, then, is when its cathode eventually fails to give sufficient emission to produce a good picture.

Life testing[1] is an invaluable aid to tube design as it enables the effect on life of new processes and techniques to be checked, as well as yielding statistical information on the performance of types in mass production.

In view of the long life of most television tubes it is not possible to run every tube tested to extinction as a tremendous number of life test positions would be required to cover even a very small sample of the manufactured product. To reduce the number of hours per tube, a removal schedule is operated so that only a small number of the tubes is left to run to extinction. By studying the variation of an emission factor, ϕ, with life it is possible to obtain an estimate of the probable emission life of the tube long before failure occurs. If there is no change in the emission factor during the period of test, it is reasonable to suppose that the life of the tube is at least that given by the average extrapolation factor. The value of ϕ is independent of tube geometry and depends merely on the quality of the cathode. It is defined by the equation :

$$I_c = 3\phi \frac{V_D^3}{V_c^{1.5}} \qquad (1)$$

where I_c is the cathode current,

V_c the cut-off voltage,

V_D the voltage from cut-off at which I_c is measured.

In a good cathode, ϕ is usually about 1·1.

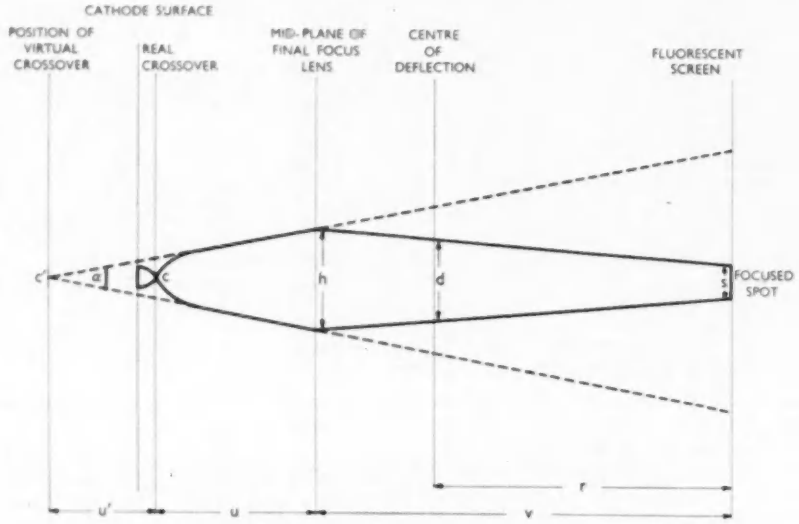

Fig. 2.—Spot formation in a television tube.

TUBE DESIGN.

The shape and path of the electron beam are shown diagrammatically in fig. 2. The electrons emitted by the cathode are attracted by the field due to the first anode, and the electron paths converge under the influence of the negative potential applied to the modulator or grid.

The circle of minimum confusion of the beam is known as the crossover. Emission from the cathode will occur so long as the ratio of the potentials applied to the first anode and modulator exceeds a certain critical value. This ratio is constant over the working range of the tube and enables the cut-off voltage for the tube at any first anode potential to be determined. In the triode cathode-ray tube (fig. 3a and b), the anode is connected to the wall coating of the tube and operates at about 14 kV. Tetrodes and pentodes (fig. 3c and d) have the first anode at a potential of about 300 V. This potential is conveniently available in the receiver, but necessitates the use of close spacings between the modulator and first anode in order to achieve the required field strength at the cathode. Modern television tubes have a cut-off voltage of about 60 to 70 volts so that the value of the ratio is about 5 for the tetrode or pentode and 200 for the triode.

In electron optics any region where the electric field is not zero constitutes an electron lens, so that in the region beyond the crossover we have a converging electron lens formed by the accelerating field of the final anode. Unlike optical lenses, the electron lens has no clearly defined boundary and the values of the refractive index may be very high indeed, so that it is unwise to pursue the optical analogy unless these features are borne in mind.

At some point in the region of the final anode the convergent effect of the lens may be assumed to have ceased, and the electrons will then travel in straight lines and produce a large, diffuse spot on the screen. If the straight path followed by these electrons is produced back towards the cathode it will be seen that the beam appears to come from a point behind the cathode. This point is the position of the virtual image of the crossover formed by the field due to the final anode. The angle at which the electron beam diverges from this point is known as the beam

angle or divergence, α, and is one of the important design parameters of the tube.

In order to reduce the size of the spot at the screen, the beam is focused by a suitable magnetic or electrostatic lens and an image of the initial crossover is formed at the screen. The electron beam normally enters and leaves this final lens at the same velocity and in this case the simple optical laws of thin lens image formation may be taken to apply. The magnification of the virtual crossover by the final lens is therefore the ratio of the distances of the image and the object from the final lens.

It would appear at first that the value of the magnification could be reduced so that the size of the final spot could be reduced indefinitely. Unfortunately this is not so, as the beam has to be deflected in order to produce a picture on the screen, and the angle through which the beam may be deflected is limited by the amount of power available to do this in the receiver. This imposes a minimum value on the image distance. There is likewise a maximum limit to the object distance imposed by the overall length of the tube, which must be as short as possible, and by the width of the beam in the focus and deflecting fields. Normally the beam width in the deflector coils is the limiting factor. If this dimension becomes excessive then the focus uniformity across the screen deteriorates owing to the non-uniformity of the field produced by the deflection coils and to the loss of focus caused by the increased distance of the edges of the screen from the centre of deflection.

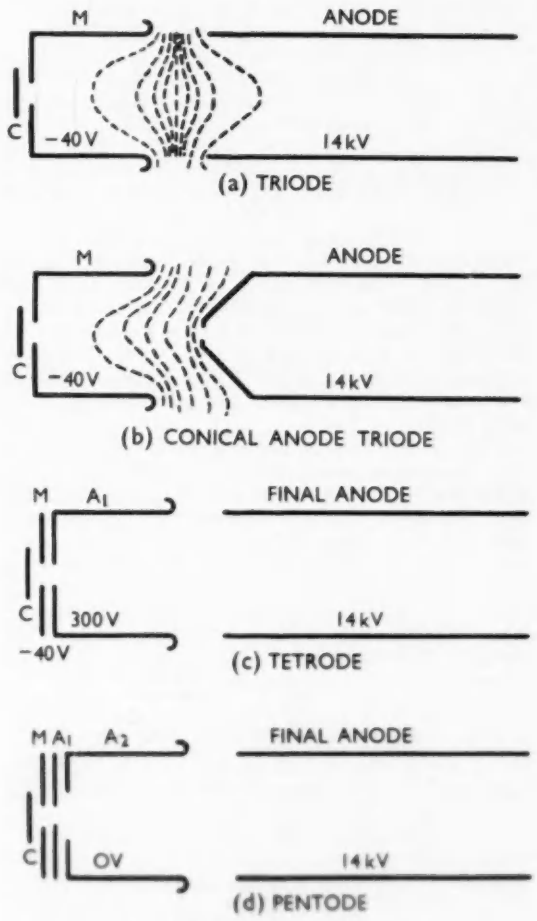

Fig. 3.—Some typical electron guns.
C—Cathode ; M—Modulator ; A_1, A_2—Anodes.

Factors Affecting the Size of the Spot.

As has been explained, the focused spot on the screen is a magnified image of the virtual crossover. It is evident therefore that so far as the final lens is concerned the significant features of the electron gun are :

(i) The size of the virtual crossover.
(ii) The position of the virtual crossover.
(iii) The beam divergence from the virtual crossover.

The method by which the appropriate characteristics are obtained is quite irrelevant from this viewpoint, and the tube may be a triode, tetrode or pentode. It is possible, however, that it may be more convenient to obtain specific values of (i), (ii) or (iii) by means of a particular type of gun.

Referring now to fig. 2, we have a diagrammatic representation of the interdependence of some of the principal dimensions of the tube. For the purpose of this work we shall make the following assumptions.

(*a*) The size of the virtual crossover c' is small compared with the beam diameter in the focus and deflection regions h and d respectively.

(*b*) The beam angle is small so that $\alpha = 2 \tan \frac{\alpha}{2}$.

Then we have by simple geometry :

$$h = (u+u')\alpha \qquad (2)$$

$$\frac{h-s}{v} = \frac{d-s}{r} \qquad (3)$$

and by the laws of optical magnification :

$$\frac{s}{c'} = \frac{v}{u+u'} \qquad (4)$$

Now from (2) and (4) :

$$\frac{s}{c'} = \frac{v\alpha}{h} \quad \therefore \; s = c'\alpha \cdot \frac{v}{h}. \qquad (5)$$

If we take d as being the limiting factor for the beam diameter, we must now use equation (3), whence :

$$\frac{h}{v} = \frac{d}{r} - s\left(\frac{1}{r} - \frac{1}{v}\right),$$

and : $$\therefore \; c'\alpha = s\left[\frac{d}{r} - s\left(\frac{1}{r} - \frac{1}{v}\right)\right]. \qquad (6)$$

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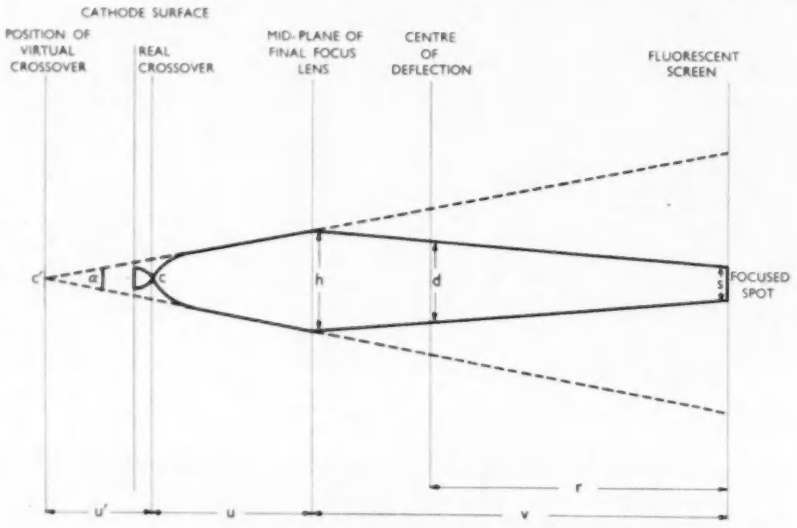

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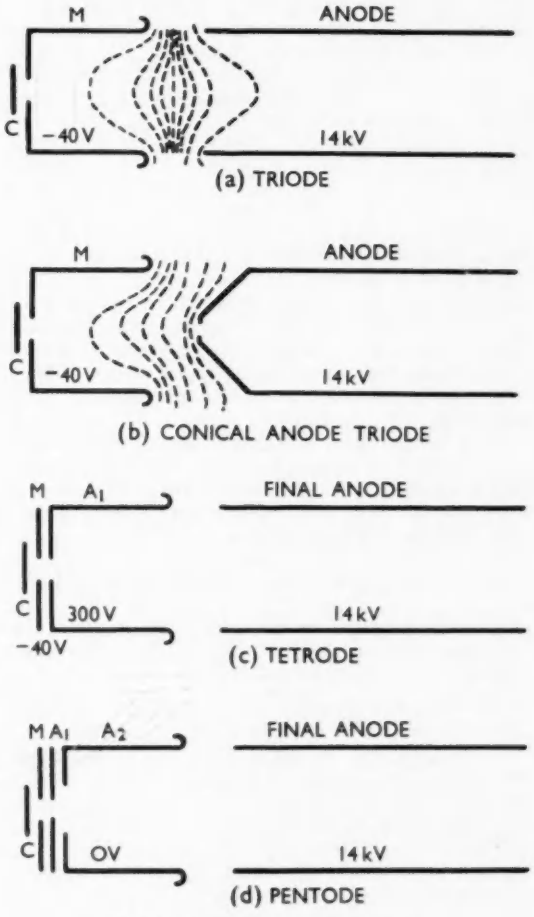

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$$\frac{h}{v} = \frac{d}{r} - s\left(\frac{1}{r} - \frac{1}{v}\right),$$

and :

$$\therefore c'\alpha = s\left\{\frac{d}{r} - s\left(\frac{1}{r} - \frac{1}{v}\right)\right\} \qquad (6)$$

Now the second term in the above expression $s\left(\frac{1}{r}-\frac{1}{v}\right)$ arises from the finite size of the final spot and its value in practice is about $\frac{1}{6}$ of $\frac{d}{r}$.

If we therefore neglect the second term we arrive at the simple expression :—

$$s=c'\alpha.\frac{r}{d}. \qquad (7)$$

The interesting feature of this expression is the independence of the various factors from each other. For example the only factors under the control of the gun designer are c' and α, and a reduction in the product $c'\alpha$ represents an improvement in performance of all tubes, irrespective of tube dimensions.

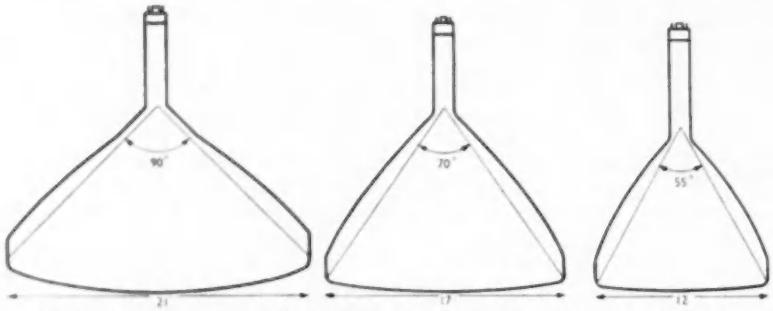

Fig. 4.—Showing how increased scanning angles have been used to restrict tube length as picture size is increased.

If we introduce the function p, defined as the ratio of the spot size to the screen diameter, then :

$$p=\frac{s}{r\tan\frac{\theta}{2}}$$

where θ is the deflection angle and

$$p=c'\alpha.\frac{1}{d\tan\frac{\theta}{2}} \qquad (8)$$

This means that the resolving power of the tube is unaltered by size so long as the deflection angle remains constant, although usually $c'\alpha$ becomes larger owing to the increased beam current required in larger tubes.

The factor $\frac{1}{d\tan\frac{\theta}{2}}$ rests largely with the deflection coil designer, but generally the set manufacturer demands the highest value of deflection angle possible in order to reduce cabinet depth and this has to be balanced against the power required to scan the tube.

The way in which increased length has been avoided as tube diameter has increased, by using larger scanning angles, is illustrated in fig. 4. We see that the tube designer may now concentrate on the value of $c'\alpha$ and attempt to arrive at the lowest value of this product consistent with reliability and certain practical requirements.

The Electron Gun Quality Factor.

Before examining this gun quality factor we may consider how it can be measured. If we consider equation (4) :

$$\frac{s}{c'}=\frac{v}{u+u'}$$

we see that s, u and v may be readily measured, but c' and u' are unknown. It is possible by removing the deflector coils to measure s in a magnetically-focused tube for a range of values of u and v, and we notice that equation (4) may be written :

$$u+u'=\frac{v}{s}c' \quad . \qquad (9)$$

If we therefore plot u against $\frac{v}{s}$, as for example in fig. 5, then, by extrapolation to $\frac{v}{s}=0$, we can find the value of u' while the slope of the line gives c'. Care must be taken to ensure that for the lower values of u, the focusing field does not penetrate the accelerating field of the final anode, otherwise errors will be introduced. In order to measure α the focusing field is removed and the diameter of the defocused spot is measured. The beam angle is then the ratio of this to $a+u'$. For many cathode-ray tubes u' is small, and a rough value of $c'\alpha$ may be obtained from a single value of u by neglecting u'.

One of the laws of electron optics which has a close parallel in optics, in which it is known as the Abbe sine law, is the law of refraction, which we may apply to the relationship between the true crossover and the virtual crossover thus :

$$c\,V_C^{\frac{1}{2}}\sin\beta=c'V_A^{\frac{1}{2}}\sin\alpha \qquad (10)$$

where β is the angle at which the beam emerges from the true crossover and V_C the crossover potential, whence :

$$c'\alpha=\left(\frac{V_C}{V_A}\right)^{\frac{1}{2}}c\sin\beta. \qquad (11)$$

By assuming that the crossover is an image of the emitting area of the cathode, we may employ the law of refraction for the conditions at the cathode, and in the ideal case we should have :—

$$c'\alpha=\left(\frac{0{\cdot}1}{V_A+0{\cdot}1}\right)^{\frac{1}{2}}d, \qquad (12)$$

where d, is the emitting diameter and it is assumed that the electrons are emitted with initial energies

of 0·1 eV at angles up to 90 deg. In practice the equation does not apply as there are defects in the crossover-forming lens and values of $c'\alpha$ of about 5 times that predicted by the above equation have been obtained.

The causes of the departure from the theoretical values of $c'\alpha$ are space charge effects and limitations in the image-forming system. Lens aberrations may of course occur anywhere in the electron optical system, but these are generally only significant in the so-called triode portion of the gun. Modern electrostatic and magnetic final lenses can be made so that they introduce little aberration.

Space charge effects are particularly apparent at higher values of cathode current, when the position of the virtual crossover tends to shift. This causes a defocusing of the spot at the screen unless steps are taken to correct the focus throughout the brightness range of the tube, the expense of which is not justified commercially.

Apart from reducing the distortions arising from lens aberrations and space charge, the factors in equation (12) which affect $c'\alpha$ are the final anode voltage V_A and the emitting diameter d_c.

The effect of increasing the final anode voltage is wholly beneficial as the luminous efficiency of the fluorescent screen also increases with final anode voltage, so that every effort is normally made to ensure that the highest practicable final anode voltage is available.

The mean current density drawn from the cathode i is given by :

$$i=\frac{4I_c}{\pi d_c^2} \qquad (13)$$

whence :

$$d_c^2=\frac{4I_c}{\pi i}$$

or :

$$d_c=2\left(\frac{I_c}{\pi i}\right)^{\frac{1}{2}} \qquad (14)$$

In order to reduce d_c we can therefore reduce the total current drawn from the cathode and increase the mean current density. The cathode current will depend on the size and brightness of the picture, the luminous efficiency of the screen and the final anode voltage as follows :

$$I_c=\frac{\text{brightness } B \times \text{area } A}{V_A \times \text{luminous efficiency } \eta} \qquad (15)$$

which illustrates the advantage of high final anode voltage and also high luminous efficiency (such as that produced by aluminising).

Modern cathode-ray tubes operate at a mean current density of about 0·1 A/cm², a figure which is sometimes exceeded in other applications where reliability is less important. It seems probable that current densities will increase with the advent of new forms of cathode and there is probably still a considerable future for higher current-density operation of oxide cathodes.

We can now summarise the four methods by which improvement in cathode-ray tube performance may be achieved. The first is in the hands of the deflector coil designer, where an increase in the permissible beam diameter for a given scanning angle will bring a proportional improvement in tube performance. The second is a reduction in the aberrations in the electron lenses in the tube, which would enable the theoretical performance to be more nearly obtained. This requires a more detailed knowledge of the properties of the beam in the cathode-modulator-first anode region than exists at present. The third is the reduction of the cathode current by means of increased screen efficiency and final anode voltage. The fourth is an increase in the current density which may safely be drawn from the cathode.

It will be seen that of these four methods, only two are in the hands of the tube designer. Both problems are quite considerable and we shall leave discussion of them and consider how we can make the best tube with the techniques at present available.

CHOICE OF BEAM ANGLE.

We shall assume for the moment that the current density limitation has been stated and attained, and that the values of the aberrations are known. We shall also assume that the tube size, screen efficiency, final anode voltage, scanning angle and deflector coil characteristics are known.

This implies that the value of $c'\alpha$ is known and we now have to consider the factors which govern the choice of beam angle for the particular application.

Beam Angle in a Magnetic Focus Tube.

In the interests of economy it is necessary for the overall length of the tube to be as small as possible,

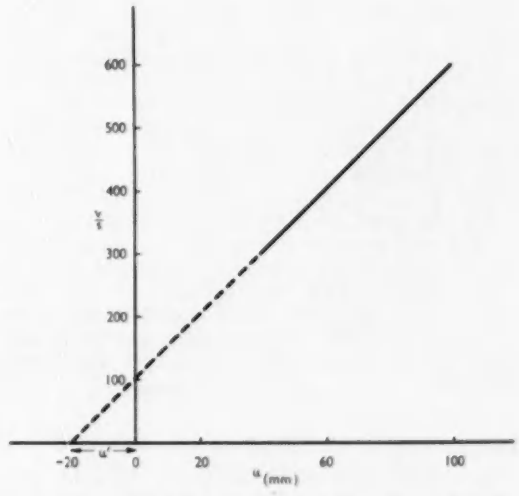

Fig. 5.—A method of determining the size and position of the virtual crossover.

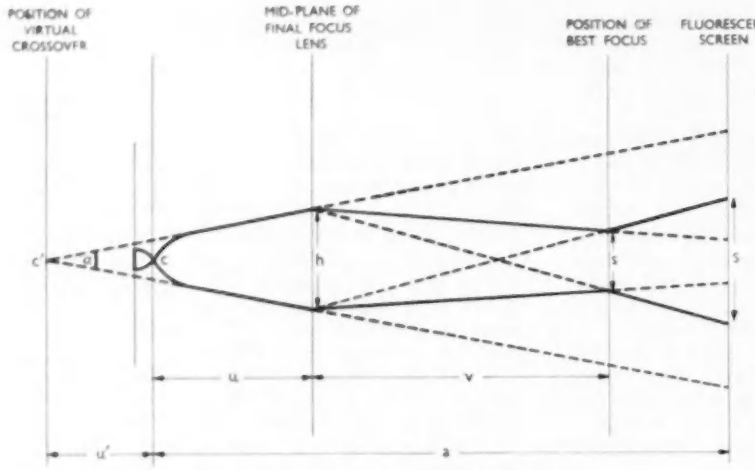

Fig. 6.—The size of a partially-defocused spot.

and, in magnetically focused tubes, for the field strength of the focus magnet to be a minimum.

The tube size and deflection angle determine the length of the bulb, and the minimum length of neck is that which is just sufficient to accommodate the deflector coils, focus magnet and, possibly, ion trap magnet without mutual interaction.

The strength of the focus magnet required may be defined as

$$F=\frac{V_A}{f} \qquad (16)$$

where f is the focal length.

Using the thin lens formula from optics we have that :

$$\frac{F}{V_A}=\frac{1}{f}=\frac{1}{v}+\frac{1}{u+u'}=\frac{1}{a-u}+\frac{1}{u+u'} \qquad (17)$$

In practice the second term has a value of about 7 times the first, so that the focus power is a minimum for maximum $(u+u')$. The value of $(u+u')$ may be made large by placing the focus magnet as far forwards as possible and by increasing the length of the tube neck. This latter method is considered undesirable in view of the necessity of keeping the overall length of the tube short, but this objection may be avoided if it is u' and not u which is increased. We therefore conclude that, in order to reduce the required strength of the magnetic lens, we employ an electron gun giving a large value of u' and a beam angle small enough for the lens to be placed as near to the deflector coils as possible without any interaction occurring.

Beam Angle in an Electrostatic Focus Tube.

Electrostatic focusing of television tubes has not yet attained the popularity in Great Britain that it has in the United States. The desirability of using readily-available sources of potential within the receiver has limited the development of electrostatic lenses in television tubes to those of the univoltage type as in fig. 7, which is a symmetrical lens having the central electrode at a low potential and the outside pair at final anode potential. The power of the lens as defined before is controlled by the electrode spacings so that there is no economic restriction attached to lens power and this is not a consideration in the design of electrostatic lenses. We will now derive an expression relating some of the design parameters with the size of the final spot for the electrostatic focus case.

If the spot at the screen is partially defocused as in fig. 6, where we assume that a focus voltage more negative than that for optimum has been applied, we may obtain the following relationships as before :

$$h=(u+u')\alpha \qquad (18)$$

$$\frac{s}{c}=\frac{v}{u+u'} \qquad (19)$$

$$\frac{S-s}{2(a-u-v)}=\frac{s+\frac{h-s}{2}}{v}=\frac{h+s}{2v}$$

whence :

$$\frac{S-s}{h+s}=\frac{a-u-v}{v}$$

$$\frac{S+h}{s+h}=\frac{a-u}{v}$$

$$\therefore\ S+h=\frac{(a-u)}{v}\left(\frac{vc'}{u+u'}+h\right)$$

which we differentiate to find how S varies if the value of v is altered without moving the lens.

$$\therefore\ \frac{dS}{dv}=-\frac{h(a-u)}{v^2} \qquad (20)$$

An expression for the focal length which has been found to apply experimentally is :

$$\frac{1}{f}=\left(\frac{V_A-pV_F}{V_A}\right)k=\frac{1}{v}+\frac{1}{u+u'} \qquad (21)$$

where p and k are constants. We differentiate to find how the value of V_F affects v and :

$$\frac{-pk}{V_A}\frac{dV_F}{dv}=\frac{-1}{v^2} \qquad (22)$$

$$\therefore\frac{dS}{dV_F}=\frac{dS}{dv}\Big/\frac{dV_F}{dv}=\frac{hpk(a-u)}{V_A} \qquad (23)$$

which tells us how V_F affects S. Now if the tube

normally focuses at $V_F=0$, then under this condition $u+v=a$ and :

$$k=\frac{1}{v}+\frac{1}{u+u'}=\frac{a+u'}{v(u+u')} \qquad (24)$$

We insert these values into equation (23) and :

$$\frac{dS}{dV_F}=\frac{hp(a+u')}{V_A(u+u')}$$

and we use equation (18) to get :

$$\frac{dS}{dV_F}=\frac{-p\alpha(a+u')}{V_A} \qquad (25)$$

Now $\alpha(a+u')$ is the size of the spot which would appear on the screen if there were no final focus lens at all, and p is found to have a value 3 for a certain design of lens. We have shown, therefore, that if we wish to make the spot size at the screen insensitive to focus voltage, then the requirements are a high final anode voltage, small beam angle and small overall length. It will be seen that these conditions are met in modern tubes having high final anode voltages, small beam angles and large deflection angles. It is therefore easier to produce a zero-voltage electrostatic focus tube which does not require a variable focus supply.

A further advantage of the zero voltage gun is that, referring to equation (21), when $V_F=0$ then $\frac{1}{f}=k$ and the focal length of the lens is constant. This means that the tube is in focus at all final anode voltages. This is not the case for a magnetic focus tube, as it defocuses if the final anode voltage changes. This may occur on scenes containing large white areas, which may cause the final anode voltage to fall.

There are several disadvantages of electrostatic focusing, the majority of the difficulties being practical rather than theoretical. The use of an electrostatic lens within the tube rather than a magnetic lens outside results in a greater liability to spherical aberration and spot-distortion. So long as the lens employed is fairly large, the limit of beam diameter is similar to that set by the deflector coils. The main requirement is to ensure that the beam is central with respect to the axis of the electrostatic lens. A slight eccentricity of the beam results in spot-distortion and incorrect focus voltage. The ion trap is commonly used as a means of centring the beam and on such tubes it will be found that the setting of the ion trap magnet has a considerable effect on spot quality. Centring of the picture with respect to the mask in the receiver must, of course, be carried out after the beam has passed through the electrostatic lens, usually with the aid of small permanent magnets.

The power of the electrostatic lens is very critical with respect to variations in its geometry, and very close tolerances must be maintained. In a typical lens a change in anode-to-anode spacing of 1 mm. is sufficient to cause a change in focus voltage of 0·1 times V_A. The lens is scarcely less sensitive to variations in the diameter of the focus electrode. It will be evident that not only have these dimensions to be accurately maintained, but the parts must be precisely circular and coaxial in order to avoid astigmatism.

The question of supporting the lens is also a problem as the number of regions of high electric field strength is trebled and consequently the risk of flashover is increased. In order to reduce this risk to a minimum, the low-voltage electrodes must be free from sharp edges and burrs. Polishing of the focus electrode by chemical or mechanical means is an advantage. Spacings between electrodes of at least 2·0 mm. avoid the necessity for unduly expensive processing of the parts.

Electrostatically-focused tubes are capable at best of giving results as good as magnetic tubes, but, apart from the independence of focus voltage of variations in the final anode voltage, there is no inherent superiority in use. The question of whether to use electrostatic focusing rather than magnetic is therefore economic rather than technical and rests between the relative costs of supplying a focus magnet and the additional tube cost incurred by the addition of the lens.

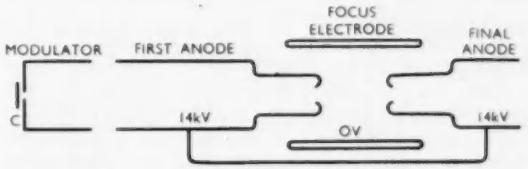

Fig. 7.—An electrostatic-focus electron gun.

PRODUCING THE DESIRED BEAM ANGLE.

We have now examined both electrostatic and magnetic focusing and have found that where the value of $c'\alpha$ is constant, then it is preferable to use values of α as low as possible consistent with using the whole of the permitted diameter in the deflector coils, and with a large value of u' also if possible.

We will now consider means of controlling the beam angle and the various types of electron gun which are employed. The obvious method of reducing α and increasing u' is to employ a second focus magnet nearer the cathode. This method suffers on the score of expense and has the technical disadvantage that the stray magnetic field is liable to cause spot-distortion in the cathode-modulator-first anode portion of the gun. Modern practice exclusively employs the electrostatic lens adjoining the final anode to produce the desired prefocusing effect.

The Triode Gun.

Let us consider first the triode gun (fig. 3a). In this gun there is no first accelerating anode and the whole of the beam accelerating is done by the final anode. Because the final anode does all the beam

accelerating, it is often believed that there is no prefocusing action. This is quite untrue, as any accelerating field does, in fact, constitute a lens, but the power of the lens does depend very much on its design. When designing a triode lens it must be borne in mind that the shape and position of the final anode and the modulator influence the cut-off voltage, so that the gun must give its required performance with a reasonable cut-off voltage. Modern television tubes generally have a cut-off voltage of about 60 to 70 volts and may require up to about 25 volts of modulation for the picture highlights.

By examination of the relation between modulator voltage and cathode current in equation (1) it will be seen that the drive voltage V_D increases as V_c increases, so that an upper limit to the cut-off voltage is set by the drive available from the receiver. A lower limit is set by the spot deterioration which occurs in low cut-off tubes. This results from the larger percentage drive, $\frac{V_D}{V_c}\cdot 100$, which is required at lower cut-off voltage. Since the spot size and drive voltage are roughly proportional, the spot size increases rapidly for tubes with lower cut-off voltages.

A common form of the triode gun is shown in fig. 3a, where it will be seen that the prefocusing action is obtained in the two-tube lens formed by the modulator cup and the anode barrel. Cupping the modulator is the chief method of obtaining prefocusing in the triode, but also has the effect of weakening the field at the cathode and so reducing the cut-off voltage.

It will be seen that the lens is almost symmetrical, having a plane of symmetry midway between the lip of the modulator cup and that of the anode barrel. The portion of the lens before this midplane is convergent and that beyond is divergent. At first sight it would appear that the net effect would be zero, but in fact the lens is convergent since the electrons are travelling slower in the convergent portion. Nevertheless, the divergent part does weaken the overall power of the lens considerably.

It is evident that if the divergent portion of the lens were eliminated, then the overall power would be increased. This can be done readily by placing an apertured disc at the end of the anode barrel facing the modulator. There is, however, a small divergent effect from the small lens at the aperture itself. This method has the advantage over deepening the modulator cup of increasing rather than decreasing the electric field at the cathode, and thereby avoiding the necessity of reducing the cathode-modulator spacing in order to maintain the correct cut-off voltage. The lens may be made stronger still by replacing the disc by an apertured cone which increases further the curvature of the equipotentials. By this means it is possible to achieve beam angles as narrow as are required in practice while yet retaining adequate field strength at the cathode.

A gun of this type is employed in the 21 in. 90 deg. scan-angle G.E.C. tube, the 7501A, where a narrow beam angle is required. In this gun, as shown in fig. 3b, the curvature of the equipotentials is still further increased by the conical shape of the anode. This gun has a beam angle of ·05 radians at zero bias, which is a suitable value to give good results with the deflection coils which are commercially available.

The Tetrode Gun.

The modification of beam angle in tetrodes is easier than with triodes since the factors governing the cut-off voltage in the tetrode are the potential of the first anode and its spacing from the modulator. The beam angle is controlled by the geometry of the first and final anodes and is therefore independent of the cut-off voltage. Unlike the triode, however, the beam angle of the tetrode is dependent on the first and final anode potentials.

During discussion of the relative merits of triode and tetrode television tubes, arguments are often put forward which are based on the relative virtues of triode and tetrode valves. This analogy is by no means valid except for the fact that the cut-off depends on the "anode" voltage in the triode and the "screen" voltage in the tetrode. So long as the cut-off voltages are the same, then the cathode current/modulator voltage characteristic is the same, i.e.

$$I_c = 3\frac{V_D^3}{V_c^{1.5}}.$$

It is often stated that the tetrode may be operated over a wider range of final anode voltages than the triode, but, while it is true that the cut-off voltage remains constant in the tetrode, this is offset to some extent by the fact that the beam angle remains constant in the triode. In order to produce the same beam angle as that of the triode it is necessary to reduce the first anode voltage in the tetrode to give the same cut-off as the triode.

We see, therefore, that there is little to choose in performance between the triode and the tetrode. The choice lies between the triode, with its greater ease of manufacture and consequent greater uniformity of performance; and the tetrode, with its necessity of accurate shape and position of the first anode and consequent liability to spot distortion, but which in practice has a somewhat greater flexibility in use.

The Pentode Gun.

A further variation which has appeared on the market is the pentode. In this tube the beam shaping is carried out between the cupped second anode and the final anode. The first anode merely controls the cut-off voltage while the potential applied to the second anode controls the beam angle. This gives the user the facility of adjusting the beam angle of the tube to achieve the best compromise between focus uniformity and central spot quality with the particular deflector coils employed. In practice little use is made of this facility and the tube is operated with the second anode at zero.

With this type of construction, where the beam shaping is carried out by a lens separate from the

crossover region, the value of u' is larger than with tetrodes and triodes. This enables a small reduction in the overall length of the tube to be made. This property is not a fundamental feature of the pentode system alone, but so far designers of tetrodes and triodes have not produced tubes having such large values of u'.

Apart from these properties the pentode is not different from the other types of tube as a picture reproducer and the beam current/modulator voltage characteristic is still governed by the same expression.

CONCLUSION.

In our discussion of electron guns we have considered the guns as having equal restrictions in current density at the cathode and in cut-off voltage. The relationships between current density and the other design parameters have been discussed fully in the literature.[2] As we have seen, it is by increasing the cathode current density that a better tube performance is obtained, and this approach, together with a better understanding of the properties and formation of the crossover, appear to be the most likely factors to improve tube performance in the future.

Electron guns for colour television have not been discussed since the various methods of picture reproduction are not sufficiently standardised for generalisations to be made concerning the gun requirements. Judging by the low luminous efficiency of the systems which have so far appeared it seems certain that higher cathode currents will be required.

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TECHNICAL LITERATURE

A digest of recent articles and papers, by members of the G.E.C. and its associated companies

THE MECHANICAL DESIGN OF BOGIE ELECTRIC LOCOMOTIVES.

By E. H. Croft (Witton Engineering Works).

The Railway Gazette, Vol. 102, *No.* 15, *pp.* 426-430, *April* 15*th*, 1955.

The course of development which has led to the general adoption to-day of bogie locomotives instead of fixed frame types is outlined, and design requirements are discussed in the light of the movements to which a locomotive is subject when travelling along the track. A simple two-bogie four-axle locomotive for high-speed and mixed traffic working is taken as the basis for study. Suggestions are made regarding body construction, its springing relative to the bogies, and the arrangement of bearers. The author gives reasons why bogie articulation for the transmission of tractive forces may be undesirable, but explains the advantages of interconnection for bogie control on curves and indicates methods of achieving it. It is suggested that the future is likely to see the fully-springborne motor adopted for all types of bogie locomotives, and that the axle-hung system would not have found such wide application had a really sound and simple connecting rod design materialised ; or had the ill effects on the track of the axle-hung motor and early bogie designs been scientifically and financially investigated.

MATRIX METHODS FOR THE EVALUATION OF SIMULTANEOUS FAULTS IN THREE-PHASE SYSTEMS.

By W. E. Lewis (College of Technology, Birmingham) and J. H. Banks (Witton Engineering Works).

I.E.E. Monograph No. 125, *April*, 1955. *Republished in Proc. I.E.E., Part C, Vol.* 102, *pp.* 231-243, *September*, 1955.

The paper illustrates two methods for the solution of a (faulted) 3-phase system when multiple unsymmetrical faults are applied.

In the first method, all faults are replaced by a balanced condition for which the symmetrical component impedance matrix is easily obtained using Stigants Rule. This matrix is then transformed to that describing the faulted network by means of a connection matrix and relationships developed by Kron.

The second method shows how Stigants Rule may be applied directly to write down the impedance matrix of the symmetrical component network representing the unsymmetrical system.

THE MEASUREMENT AND DISPLAY OF THE CURRENT GAIN (α) OF A TRANSISTOR AS A FUNCTION OF EMITTER CURRENT (607).*

By R. A. Hilbourne (Research Laboratories).

Journal of Scientific Instruments, Vol. 32, *pp.* 83-85, *March*, 1955.

An apparatus for displaying, on a cathode-ray oscilloscope, the variation of the current gain α of a point-contact transistor is described. The design of this apparatus is discussed with particular reference to the amplifier bandwidth requirements that are necessary to display accurately the rapid variations in α. The displayed curves for a typical point-contact transistor over various ranges of emitter current are given.

ELECTRODE SHAPES FOR A CYLINDRICAL ELECTRON BEAM (609).*

By P. N. Daykin (Research Laboratories).

British Journal of Applied Physics, Vol. 6, *pp.* 248-250, *July*, 1955.

The problem of obtaining the electrode shapes which will maintain a cylindrical beam of space-charge limited current reduces to the problem of solving Laplace's equation for the electrostatic potential outside the space-charge region, with the appropriate boundary conditions on the surface of the beam. A new approximate analytical solution of the Laplace equation has been obtained, and numerical tables have been developed for designing the electrode shapes. A plot of the electrode shapes, which is applicable to either hollow or solid beams, has been drawn up. The shape of the outside electrodes is the same for either type of beam, but the hollow beam requires an additional set of electrodes located inside the beam. The shape of the outside electrodes agrees fairly well with that obtained previously by Pierce, Harrison, and Kuo Chu Ho and Moon.

BREAKDOWN EFFECT IN p-n ALLOY GERMANIUM JUNCTIONS (616).*

By R. D. Knott, I. D. Colson and M. R. P. Young (Research Laboratories).

Proceedings of the Physical Society, B, Vol. 68, *p.* 182, *March*, 1955.

As a result of the early measurements of McAfee and others (1951) and the later work of McKay and McAfee (1953) two forms of breakdown mechanism have been recognised in grown germanium junctions, namely an avalanche breakdown in broad junctions and internal field emission in junctions which are too narrow to support appreciable multiplication. In the present work these processes have been examined in connection with indium bead type germanium *p-n* alloy junctions. Slices of single-crystal *n*-type germanium, with known resistivities in the range 0·2-4 ohm. cm., were cut perpendicular to the 110 direction and eight junctions, approximately 1 mm. in diameter, were made on each slice using the well-established alloying technique with indium as the *p*-type impurity. The breakdown potentials were determined by applying an adjustable 50 c/s. A.C. voltage across the junctions and suitable series resistors and displaying the reverse current-voltage characteristics on a calibrated cathode-ray oscilloscope. A variation of some 20 per cent. in the value of the breakdown voltage was found over a batch of normal junctions made on base material of a given resistivity and, in all cases, the maximum observed value was accepted as that most probably associated with the quoted resistivity.

The good agreement between the predicted and observed values indicates that the above theory may be equally well applied to alloy type as to grown germanium *p-n* junctions.

CHANGES IN THE STRUCTURE OF OXIDE CATHODES AT HIGH TEMPERATURES (629).*

By H. P. Rooksby (Research Laboratories).

British Journal of Applied Physics, Vol. 6, *pp.* 272-276, *August*, 1955.

The effect of heat treatment on barium oxide and strontium oxide thermionic cathodes has been examined by X-ray diffraction methods. Crystal growth in these cathodes has been studied by a simple microbeam technique. It has been shown that in the temperature range 900-1,150 deg. C., crystallites of barium oxide between 2 and 5 μ in size are normally developed. This size is considerably greater than had been estimated previously by X-ray line broadening methods. Similar sizes develop in strontium oxide in the temperature range 1,100-1,250 deg. C.

It has also been found that in both oxides a gradual decrease in the structure-cell dimensions takes place with increasing temperature of heating in a vacuum. This change is attributed to a progressive increase in the numbers of defects in the lattice and, in the case of barium oxide, appears to correspond with the process of thermionic activation.

THE LIGHTING OF UNDERGROUND ROADWAYS IN COLLIERIES.

By H. F. Stephenson and W. R. Stevens (Research Laboratories).

The Colliery Guardian, Vol. 190, *No.* 4,910, *pp.* 437-443, *April 7th*, 1955.

The major problem of providing sufficient light without glare on colliery roadways was studied in controlled laboratory conditions by the construction of a model roadway to half scale. Experiments were made to determine a satisfactory light distribution from a well glass fitting, and were followed by full-scale experiments in a pit. The article describes the subsequent design of practical flameproof and non-flameproof fittings with the desired characteristics and the tests carried out both in the model roadway and at a colliery. It is considered that the use of the fittings developed as a result of this investigation should allow a miner to enter a well-lit shaft bottom and proceed safely to his place of work along an adequately lighted road.

TRANSPARENT CHARTS FOR SOLVING ENGINEERING PROBLEMS.

By B. B. Daly (Woods of Colchester Ltd.).

The Engineer, Vol. 199, *No.* 5186, *pp.* 841-844, *June 17th*, 1955.

A system of charts and transparencies is described from which the solutions of problems involving up to five independent variables may be read directly. The method, in which a transparent " cursor " sheet is located over a base chart, was devised to assist in the selection of propeller, and later adjustable pitch axial flow fans. In this article the construction of the fan selection chart is described, some general observations are put forward on the principles and limitations of the system, and a new chart is described for the solution of the well-known Colebrook-White pipe friction equation. It is suggested that the system is particularly adapted to fluid flow problems where arbitrary relations between velocity, size, pressure, drag, sound level, heat transfer and other variables are expressed with dimensionless factors involving products and powers of the variables. Experience has shown that the initial trouble of producing a chart and a few cursors is well repaid by the facility with which a wide field of size, speed, etc., can be explored.

WINDING ENGINE CALCULATIONS FOR THE MINING ENGINEER.

By A. B. Price (Fraser & Chalmers Engineering Works).

Second, revised edition, published by the Fraser & Chalmers Engineering Works of The General Electric Co. Ltd., 1955. *Price* 30*s*.

Originally published in 1951, this book has since been revised to include more details of friction winders, both Koepe and multi-rope types. The revision maintains the unique status of the book as a guide to engineers within the mining industry in planning the best and most economical installations.

The complexities of design are discussed at length with particular reference to the problems of the site engineer, and an effort has been made to deal with practice both in the United Kingdom and overseas.

All systems at present practised are noted and a chapter on control schemes is useful in orientating the calculations to particular problems. For drum winders, calculations are given relating to cylindrical drums, B.C.C. drums and double drums ; in dealing with friction winders, Koepe wheels and multi-rope winders are fully described.

A chapter on inclined shafts considers the mechanics of this method. Calculations for steam winders form an end piece to the book. Useful tables are included on ropes ; also on moments of inertia for typical A.C. and D.C. motors, drums and reduction gears.

The book is extensively illustrated in line and tone, including a number of horsepower-time diagrams and five typical arrangement drawings of winders in current use.

The Company has donated a number of copies to the Electrical Industries Benevolent Association, of 10, Buckingham Palace Gardens, London, S.W.1, to whom application may be made for copies.

***A limited number of reprints is available of those papers marked with an asterisk. Copies may be obtained on application to the Editor, G.E.C. Journal, Magnet House, Kingsway, W.C.2.**